Modal effect types

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Joint work with
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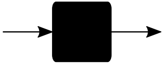
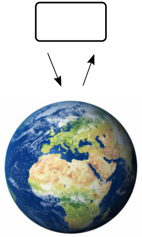
Effects

Programs as black boxes (Church-Turing model)?



Effects

Programs must interact with their environment



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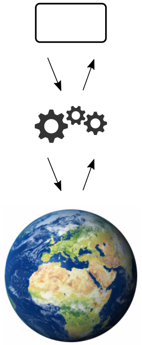

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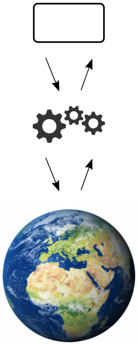
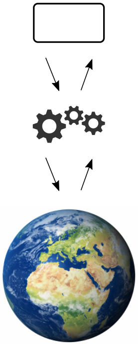


Effects are pervasive

- input/output
  user interaction
- concurrency
  web applications
- distribution
  cloud computing
- exceptions
  fault tolerance
- choice
  backtracking search

Effects

Programs must interact with their environment



Effects are pervasive

- input/output
  user interaction
- concurrency
  web applications
- distribution
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Effect type systems statically track the use of effects

Conventional effect types

Pure computation

```
inc : Int → Int
inc i = i + 1

app : (Int → Int) → Int → Int
app f x = f x
```

```
> app inc 42
43 : Int
```

Conventional effect types

A variant of `inc` using a `Read` effect supporting effectful operation `ask : 1 ↠ Int`

```
inc : Int → Int
inc i = i + do ask ()
```

```
app : (Int → Int) → Int → Int
app f x = f x
```

Conventional effect types

Effects are tracked statically by adding effect annotations to arrows

```
inc : Int −Read→ Int
inc i = i + do ask ()

app : (Int → Int) → Int → Int
app f x = f x
```

Conventional effect types

Effect polymorphism allows app to be used in the presence of arbitrary effects

```
inc : Int -Read-> Int
inc i = i + do ask ()

app : ∀ e.(Int -e-> Int) -e-> Int -e-> Int
app f x = f x

appinc : Int -Read-> Int ! Read
appinc = app inc
```

$\texttt{inc : Int} \xrightarrow{\texttt{Read}} \texttt{Int}$

$\texttt{app} : \forall\ \texttt{e.(Int} \xrightarrow{\texttt{e}} \texttt{Int)} \xrightarrow{\texttt{e}} \texttt{Int} \xrightarrow{\texttt{e}} \texttt{Int}$

$\texttt{appinc : Int} \xrightarrow{\texttt{Read}} \texttt{Int ! Read}$

Conventional effect types

Effect polymorphism also allows `inc` to be used in contexts that have additional effects

$$\texttt{inc} : \forall\ \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read, e}} \texttt{Int}$$

```
inc i = i + do ask ()
```

$$\texttt{app} : \forall\ \texttt{e}.(\texttt{Int} \xrightarrow{\texttt{e}} \texttt{Int}) \xrightarrow{\texttt{e}} \texttt{Int} \xrightarrow{\texttt{e}} \texttt{Int}$$

```
app f x = f x
```

$$\texttt{princ} : \forall\ \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read, IO, e}} \texttt{Int}$$

```
princ i = do print "incrementing"; inc i
```

Conventional effect types

Effect polymorphism tracks a handler consuming an effect

$$\texttt{inc} : \forall\, \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read, e}} \texttt{Int}$$

```
inc i = i + do ask ()
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$$\texttt{app} : \forall\, \texttt{e}.(\texttt{Int} \xrightarrow{\texttt{e}} \texttt{Int}) \xrightarrow{\texttt{e}} \texttt{Int} \xrightarrow{\texttt{e}} \texttt{Int}$$

```
app f x = f x
```

$$\texttt{two} : \forall\, \texttt{e}.(1 \xrightarrow{\texttt{Read, e}} \texttt{Int}) \xrightarrow{\texttt{e}} \texttt{Int}$$

```
two f = handle f () with {ask () r ⇒ r 2}
```

```
> two (fun () → app inc 42)
44 : Int
```

Conventional effect types

$$\texttt{inc} : \forall\ \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read, e}} \texttt{Int}$$

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Conventional effect types

$$\mathtt{inc} : \forall\ \mathtt{e}.\mathtt{Int} \xrightarrow{\mathtt{Read,\ e}} \mathtt{Int}$$

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Do we really need all of these effect variables?

The **eee** problem [Martin Odersky, Vianna, March 2026]

Conventional effect types

$$\texttt{inc} : \forall\ \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read, e}} \texttt{Int}$$

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Do we need effect polymorphism at all?

Conventional effect types

$$\texttt{inc} : \forall\ \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read, e}} \texttt{Int}$$
$$\texttt{princ} : \forall\ \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read, IO, e}} \texttt{Int}$$
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The **eee** problem [Martin Odersky, Vianna, March 2026]

Do we need effect polymorphism at all?

Can we add effect types to existing languages without rewriting type signatures?

Conventional effect types

$$\texttt{inc} : \forall\, \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read, e}} \texttt{Int}$$

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Conventional effect types

$$\texttt{inc} : \forall\, \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read, e}} \texttt{Int}$$
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Do we really need all of these effect variables? No!

The **eee** problem [Martin Odersky, Vianna, March 2026]

Do we need effect polymorphism at all? Rarely

Can we add effect types to existing languages without rewriting type signatures?

Conventional effect types

$$\texttt{inc} : \forall\, \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read, e}} \texttt{Int}$$

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Do we really need all of these effect variables? No!

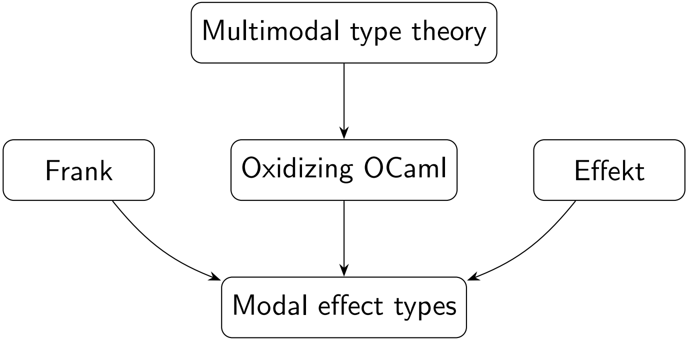
The **eee** problem [Martin Odersky, Vianna, March 2026]

Do we need effect polymorphism at all? Rarely

Can we add effect types to existing languages without rewriting type signatures? Maybe

Modal effect types

Inspiration



Key ideas

- **decouple** effect typing from function arrows
- track effects through an **ambient effect context**
- use **modalities** to modify the ambient effect context locally

Modal effect types

Pure computation

```
inc : Int → Int
inc i = i + 1

app : (Int → Int) → Int → Int
app f x = f x
```

```
> app inc 42
43 : Int
```

Modal effect types

A variant of `inc` using a `Read` effect supporting effectful operation `ask` : $1 \twoheadrightarrow$ `Int`

```
inc : Int → Int
inc i = i + do ask ()
```

```
app : (Int → Int) → Int → Int
app f x = f x
```

Modal effect types

Effects (and purity) are tracked statically using **absolute modalities**

```
inc : [Read](Int → Int)
inc i = i + do ask ()
```

```
app : []((Int → Int) → Int → Int)
app f x = f x
```

Modal effect types

Subeffecting allows `app` to be used in the presence of arbitary effects

```
inc : [Read](Int → Int)
inc i = i + do ask ()

app : []((Int → Int) → Int → Int)
app f x = f x

appinc : [Read](Int → Int)
appinc = app inc
```

Modal effect types

Subeffecting also allows `inc` to be used in contexts that have other effects too

```
inc : [Read](Int → Int)
inc i = i + do ask ()

app : []((Int → Int) → Int → Int)
app f x = f x

princ : [Read, IO](Int → Int)
princ i = do print "incrementing"; inc i
```

Modal effect types

Relative modalities track a handler consuming an effect

```
inc : [Read](Int → Int)
inc i = i + do ask ()

app : []((Int → Int) → Int → Int)
app f x = f x

two : []((<Read>(1 → Int)) → Int)
two f = handle f () with {ask () r ⇒ r 2}
```

```
> two (fun () → app inc 42)
44 : Int
```

Comparing conventional effect types with modal effect types

Conventional effect types

$$\texttt{inc} : \forall\, \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read},\ \texttt{e}} \texttt{Int}$$

$$\texttt{princ} : \forall\, \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read},\ \texttt{IO},\ \texttt{e}} \texttt{Int}$$

$$\texttt{app} : \forall\, \texttt{e}.(\texttt{Int} \xrightarrow{\texttt{e}} \texttt{Int}) \xrightarrow{\texttt{e}} \texttt{Int} \xrightarrow{\texttt{e}} \texttt{Int}$$

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Modal effect types

$$\texttt{inc} : [\texttt{Read}](\texttt{Int} \to \texttt{Int})$$

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$$\texttt{two} : [\,](\langle\texttt{Read}\rangle(\texttt{1} \to \texttt{Int}) \to \texttt{Int})$$

Comparing conventional effect types with modal effect types

Conventional effect types

$$\texttt{inc} : \forall\ \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read, e}} \texttt{Int}$$

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Modal effect types

$$\texttt{inc} : [\texttt{Read}](\texttt{Int} \to \texttt{Int})$$

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$$\texttt{app} : (\texttt{Int} \to \texttt{Int}) \to \texttt{Int} \to \texttt{Int}$$

$$\texttt{two} : \langle\texttt{Read}\rangle(1 \to \texttt{Int}) \to \texttt{Int}$$

(we allow top-level empty absolute modalities to be omitted)

Comparing conventional effect types with modal effect types

Conventional effect types

$\texttt{inc} : \forall\, \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read, e}} \texttt{Int}$

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Modal effect types

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Modal effect types solve the eee problem!

Modal effect typing

A **mode** is an effect context

A **modality** is a transformation from one mode to another

MET — simply-typed core calculus of modal effect types

METL — surface language for MET with: bidirectional typing for inferring introduction and elimination of modalities + algebraic data types + polymorphism

Almost all examples in these slides use the **simply-typed** fragment of METL

Prototype implementation of METL:

https://github.com/thwfhk/met-oopsla25-artifact/tree/main

From function arrows to effect contexts

Conventional effect typing — function arrows are annotated with effects

$$\vdash \ \ \texttt{fun (f, x)} \to \texttt{f x} : ((\texttt{Int} \xrightarrow{\texttt{E}} \texttt{1}) \times \texttt{Int}) \xrightarrow{\texttt{E}} \texttt{1} \ \ \texttt{!} \ \ .$$

From function arrows to effect contexts

Conventional effect typing — function arrows are annotated with effects

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Modal effect typing — **ambient effect context** determines effects

$\vdash\ \ \texttt{fun (}\underbrace{\texttt{f}}_{\texttt{@ E}}\texttt{, x)}\ \underbrace{\to \texttt{f x}}_{\texttt{@ E}} : ((\underbrace{\texttt{Int} \to \texttt{1}}_{\texttt{@ E}}) \times \texttt{Int})\ \underbrace{\to \texttt{1}}_{\texttt{@ E}}\ \ \texttt{@}\ \ \texttt{E}$

Effect contexts

An **effect** is a set of effecful operations. Examples:

```
effect Read = ask:1 ↠ Int
effect IO = print:String ↠ 1
effect State a = get:1 ↠ a, put:a ↠ 1
effect Gen a = yield:a ↠ 1
```

An **effect context** `E` is a row of effects

Effect contexts

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```

An **effect context** `E` is a row of effects

Effect context rows are **scoped** (as in Frank and Koka)

- repeats are allowed (same name but possibly operation signatures)
- order of repeated effects matters
- relative order of distinct effects does not matter

Overriding the ambient context with absolute modalities

$$\vdash\ \texttt{fun x} \rightarrow \underbrace{\texttt{do yield (x + 42)}}_{\texttt{@ Gen Int}}\ \texttt{:}\ (\underbrace{\texttt{Int} \rightarrow \texttt{1}}_{\texttt{@ Gen Int}})\ \texttt{@ Gen Int}$$

Overriding the ambient context with absolute modalities

$$\vdash\ \texttt{fun x} \to \underbrace{\texttt{do yield (x + 42)}}_{\texttt{@ Gen Int}} : (\underbrace{\texttt{Int} \to \texttt{1}}_{\texttt{@ Gen Int}})\ \texttt{@ Gen Int}$$

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The **absolute modality** `[Gen Int]` **overrides** the empty ambient effect context (.) in the function body enabling the `yield` operation to be performed.

Overriding the ambient context with absolute modalities

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In general `[E]` overrides the ambient effect context with `E`.

Overriding the ambient context with absolute modalities

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Effect contexts given by absolute modalities **percolate** through the structure of a type:

- a function of type `[E](A → B)` may perform effects `E` when invoked
- elements of a list of type `[E](List (A → B))` may perform effects `E` when invoked
- a value of type `[E]Int` cannot perform any effects

Absolute modalities and higher-order functions

Iteration specialised to integer lists:

```
iter : [](((Int → 1) → List Int → 1)
iter f nil           = ()
iter f (cons x xs) = f x; iter f xs
```

Absolute modalities and higher-order functions

Iteration specialised to integer lists:

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iter : []((Int → 1) → List Int → 1)
iter f nil           = ()
iter f (cons x xs) = f x; iter f xs
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Applying a pure higher-order function in an impure effect context:

```
⊢  iter (fun x → do yield (x + 42)) : 1 @ Gen Int
```

Absolute modalities and higher-order functions

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What happened? Bidirectional typing eliminates the `[]` modality of `iter` and upcasts the empty effect context to the singleton effect context `Gen Int`.

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Terminology:

- **boxing** = modality introduction
- **unboxing** = modality elimination

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Terminology:

- **boxing** = modality introduction
- **unboxing** = modality elimination

In a conventional effect type system `iter` would be effect-polymorphic

$$\texttt{iter} : \forall\, \texttt{e}.(\texttt{Int} \xrightarrow{\texttt{e}} \texttt{1}) \xrightarrow{\texttt{e}} \texttt{List Int} \xrightarrow{\texttt{e}} \texttt{1}$$

Transforming the ambient context with relative modalities

Handling the `Gen Int` effect to produce a list of integers:

```
asList : [](<Gen Int>(1 → 1) → List Int)
asList f =
  handle f () with
    return ()  ⇒  nil
    yield x r  ⇒  cons x (r ())
```

Transforming the ambient context with relative modalities

Handling the Gen Int effect to produce a list of integers:

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asList f =
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```

In the body of asList, **relative modality** <Gen Int> **extends** the ambient effect context.

$$\vdash \mathtt{fun}\ \underbrace{\ \mathtt{f}\ }_{@\ \mathtt{Gen\ Int,\ E}} \to \mathtt{handle}\ \underbrace{\ \mathtt{f\ ()}\ }_{@\ \mathtt{Gen\ Int,\ E}}\ \mathtt{with}\ \ldots : \mathtt{<Gen\ Int>(}\ \underbrace{\mathtt{1} \to \mathtt{1}}_{@\ \mathtt{Gen\ Int,\ E}}\ \mathtt{)} \to \mathtt{List\ Int}\ @\ \mathtt{E}$$

The effect context of f is Gen Int, E.

Transforming the ambient context with relative modalities

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In the body of `asList`, **relative modality** `<Gen Int>` **extends** the ambient effect context.

$\vdash \texttt{fun}\ \underbrace{\texttt{f}}_{\texttt{@ Gen Int, E}} \to \texttt{handle}\ \underbrace{\texttt{f ()}}_{\texttt{@ Gen Int, E}}\ \texttt{with} \ldots : \texttt{<Gen Int>(}\underbrace{\texttt{1} \to \texttt{1}}_{\texttt{@ Gen Int, E}}\texttt{)} \to \texttt{List Int @ E}$

The effect context of `f` is `Gen Int, E`.

In a conventional effect type system `asList` would be effect-polymorphic

$\texttt{asList} : \forall\ \texttt{e.(1} \xrightarrow{\texttt{Gen Int, e}} \texttt{1)} \xrightarrow{\texttt{e}} \texttt{List Int}$

Overview of $\textsc{Met}$

Types	$A ::= 1 \mid A \times B \mid A \to B \mid \mu A \mid \ldots$
Signatures	$\Sigma ::= \cdot \mid \ell : A \twoheadrightarrow B$
Modalities	$\mu ::= [E] \mid \langle D \rangle$
Effect contexts	$D, E ::= \cdot \mid \ell, E$
Contexts	$\Gamma ::= \cdot \mid \Gamma, x :_{\mu} A \mid \Gamma, \text{🔒}_{\mu}$

Applying and composing modalities

Application

$$[E](F) = E$$
$$\langle D\rangle(F) = D, F$$

Composition

$$\mu \circ [E] = [E] \qquad [E] \circ \langle D\rangle = [D, E] \qquad \langle D_1\rangle \circ \langle D_2\rangle = \langle D_2, D_1\rangle$$

$$(\mu \circ \nu)(E) = \nu(\mu(E))$$

Contexts identified up to elimination of trivial locks and composition of adjacent locks

$$\Gamma, \text{🔒}_{\langle\rangle}, \Gamma' = \Gamma, \Gamma'$$
$$\Gamma, \text{🔒}_{\mu}, \text{🔒}_{\nu}, \Gamma' = \Gamma, \text{🔒}_{\mu\circ\nu}, \Gamma'$$

Locks determine available effects

$$\begin{aligned}
\mathsf{locks} &: \mathsf{Ctx} \to \mathsf{Modality} \\
\mathsf{locks}(\cdot) &= \langle\rangle \\
\mathsf{locks}(\Gamma, x :_{\mu} A) &= \mathsf{locks}(\Gamma) \\
\mathsf{locks}(\Gamma, \text{🔒}_{\mu}) &= \mathsf{locks}(\Gamma) \circ \mu
\end{aligned}$$

$$\begin{aligned}
\mathsf{effects} &: \mathsf{Ctx} \to \mathsf{EffCtx} \\
\mathsf{effects}(\Gamma) &= (\mathsf{locks}(\Gamma))(\cdot)
\end{aligned}$$

Typing rules

$$\textsc{T-Var}\quad \frac{\vdash_A \mu \Rightarrow \mathsf{locks}(\Gamma') \mathbin{@} \mathsf{effects}(\Gamma)}{\Gamma, x :_\mu A, \Gamma' \vdash x : A}$$

$$\textsc{T-Abs}\quad \frac{\Gamma, x :_{\langle\rangle} A \vdash M : B}{\Gamma \vdash \lambda x^A . M : A \to B} \qquad \textsc{T-App}\quad \frac{\Gamma \vdash M : A \to B \qquad \Gamma \vdash N : A}{\Gamma \vdash M\ N : B}$$

$$\textsc{T-Mod}\quad \frac{\Gamma, \text{🔒}_\mu \vdash V : A}{\Gamma \vdash \mathbf{mod}_\mu V : \mu A} \qquad \textsc{T-LetMod}\quad \frac{\Gamma \vdash V : \mu A \qquad \Gamma, x :_\mu A \vdash M : B}{\Gamma \vdash \mathbf{let}\ \mathbf{mod}_\mu\ x = V\ \mathbf{in}\ M : B}$$

Typing rules

$$\textsc{T-Do}\quad \frac{\Sigma \ni \ell : A \twoheadrightarrow B \qquad \mathsf{effects}(\Gamma) = \ell, E \qquad \Gamma \vdash N : A}{\Gamma \vdash \mathbf{do}\ \ell\ N : B}$$

$$\textsc{T-Handle}\quad \frac{\begin{array}{c}\Sigma \ni \ell : A' \twoheadrightarrow B' \\ \Gamma, \text{🔒}_{\langle \ell \rangle} \vdash M : A \\ \Gamma, x :_{\langle \ell \rangle} A \vdash N : B \\ \Gamma, p :_{\langle\rangle} A', r :_{\langle\rangle} B' \to B \vdash N' : B\end{array}}{\Gamma \vdash \mathbf{handle}\ M\ \mathbf{with}\ \{\mathbf{return}\ x \mapsto N;\ \ell\ p\ r \mapsto N'\} : B}$$

Kinding rules

$$\frac{}{\vdash 1 : \mathsf{Abs}} \qquad \frac{\vdash A_1 : K_1 \qquad \vdash A_2 : K_2}{\vdash A_1 \times A_2 : K_1 \sqcup K_2} \qquad \frac{\vdash A_1 : K_1 \qquad \vdash A_2 : K_2}{\vdash A_1 \to A_2 : \mathsf{Any}}$$

$$\frac{\vdash A : K}{\vdash [E]A : \mathsf{Abs}} \qquad \frac{\vdash A : K}{\vdash \langle D \rangle A : \mathsf{Any}}$$

Subkinding

$$\mathsf{Abs} \leq \mathsf{Any}$$

A type has absolute kind iff all function types or relative modalities are guarded by an absolute modality.

Promoting modalities

$$\frac{E, E' = \mu(F)}{\vdash_A [E] \Rightarrow \mu \; \texttt{@} \; F} \qquad \frac{\vdash A : \mathsf{Abs}}{\vdash_A \mu \Rightarrow \nu \; \texttt{@} \; E}$$

Coercions between modalities

Automatic unboxing in METL allows values to be coerced between different modalities

We can extend an absolute modality:

$\vdash$ `fun f` $\to$ `f : [Gen Int](1` $\to$ `1)` $\to$ `[Gen Int, Gen String](1` $\to$ `1) @ E`

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In a conventional effect type system this corresponds to:

$\vdash \texttt{fun f} \rightarrow \texttt{f : (}\forall\ \texttt{e.1} \xrightarrow{\texttt{Gen Int, e}} \texttt{1)} \xrightarrow{\texttt{E}} \texttt{(}\forall\ \texttt{e.1} \xrightarrow{\texttt{Gen Int, Gen String, e}} \texttt{1)}$

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We cannot extend a relative modality in the same way:

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$$\vdash \texttt{fun f} \to \texttt{f} : (\forall\ \texttt{e}.\texttt{1} \xrightarrow{\texttt{Gen Int, e}} \texttt{1}) \xrightarrow{\texttt{E}} (\forall\ \texttt{e}.\texttt{1} \xrightarrow{\texttt{Gen Int, Gen String, e}} \texttt{1})$$

We cannot extend a relative modality in the same way:

$\nvdash$ `fun f -> f : <>(1 -> 1) -> <Gen Int>(1 -> 1) @ E  # Ill-typed`

This would extend `E` with a `Gen Int` effect which may shadow other `Gen` effects in `E`.

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Storing effectful functions

First-order cooperative concurrency effect

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effect Coop = suspend:1 ↠ 1, ufork:1 ↠ Bool
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Recursive data type of cooperative processes

```
data Proc = proc (List Proc → 1)

push : [](Proc → List Proc → List Proc)
push x xs = xs ++ cons x nil
```

```
next : [](List Proc → 1)
next q = case q of
  nil                → ()
  cons (proc p) ps → p ps
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Scheduler parameterised by a list of suspended processes

```
schedule : [](<Coop>(1 → 1) → List Proc → 1)
schedule m = handle m () with
  return ()      ⇒  fun q → next q
  suspend () r   ⇒  fun q → next (push (proc (r ())) q)
  ufork () r     ⇒  fun q → r true (push (proc (r false)) q)
```

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```

In a conventional effect system storing effectful functions requires effect polymorphism

$$\texttt{data Proc e = proc (List Proc} \xrightarrow{\texttt{e}} \texttt{1)}$$

$$\texttt{schedule : } \forall\, \texttt{e.(1} \xrightarrow{\texttt{Coop, e}} \texttt{1)} \xrightarrow{\texttt{e}} \texttt{List (Proc e)} \xrightarrow{\texttt{e}} \texttt{1}$$

Composing handlers

State effect

```
effect State s = get:1 ↠ s, put:s ↠ 1
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effect State s = get:1 ↠ s, put:s ↠ 1
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A state handler (specialised to integer state)

```
state : [](<State Int>(1 → 1) → Int → 1)
state m = handle m () with
  return x  ⇒  fun s → x
  get () r  ⇒  fun s → r s s
  put s' r  ⇒  fun s → r () s'
```

Composing handlers

Using integer state to write a generator that yields the prefix sum of a list

```
prefixSum : [Gen Int, State Int](List Int → 1)
prefixSum xs = iter (fun x → do put (do get () + x); do yield (do get ())) xs
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We can now handle the operations of `prefixSum` by composing two handlers

```
> asList (fun () → state (fun () → prefixSum [3,1,4,1,5,9]) 0)
# [3,4,8,9,14,23] : List Int
```

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In a conventional effect system composing handlers requires effect polymorphism

$$\texttt{asList} : \forall\, \texttt{e}.(1 \xrightarrow{\texttt{Gen Int, e}} 1) \xrightarrow{\texttt{e}} \texttt{List Int}$$

$$\texttt{state} : \forall\, \texttt{e}.(1 \xrightarrow{\texttt{State Int, e}} 1) \xrightarrow{\texttt{e}} \texttt{Int} \xrightarrow{\texttt{e}} 1$$

Kinds

State handler for `1` $\to$ `1` computations

```
state' : [](<State Int>(1 → (1 → 1)) → Int → (1 → 1))
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`state` versus `state'`:

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state' : [](<State Int>(1 → (1 → 1)) → Int → <State Int>(1 → 1))
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`state` versus `state'`:

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state  : [](<State Int>(1 → 1)        → Int → 1)
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```

- `state` cannot leak the state effect
- `state'` can leak the state effect

Kinds

- **Absolute types** (e.g. `1`, `List Int`, and `[Gen Int](List Int → 1)`)
  built from base types, positive types, and types boxed by an absolute modality —
  cannot leak effects
- **Unrestricted types** (e.g. `1 → 1`, `List Int → 1`, and `<Coop>(1 → 1)`)
  also include functions not boxed by an absolute modality —
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Kinds

- `Abs` classifies absolute types
- `Any` classifies unrestricted types

Subkinding allows absolute types to be treated as unrestricted: `Abs` $\leq$ `Any`

Type polymorphism

Polymorphic version of `iter`

```
iter : ∀(a:Any).[]((a → 1) → List a → 1)
iter {a:Any} f nil          = ()
iter {a:Any} f (cons x xs) = f x; iter {a} f xs
```

Explicit type abstractions and type applications in braces.

Type polymorphism

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Explicit type abstractions and type applications in braces.

Two possible polymorphic types for handling state

```
state  : ∀(a:Abs).[](<State Int>(1 → a) → Int → a)
state' : ∀(a:Any).[](<State Int>(1 → a) → Int → <State Int>a)
```

- `∀(a:Abs)` ascribes kind `Abs` to `a`, allowing values of type `a` to escape the handler.
- `∀(a:Any)` ascribes kind `Any` to `a`, not allowing values of type `a` to escape the handler.

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Two possible polymorphic types for handling state

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- `∀(a:Any)` ascribes kind `Any` to `a`, not allowing values of type `a` to escape the handler.

Using η-expansion we can coerce `state'` to have the type of `state`

```
⊢fun {a:Abs} m s → state' {a} m s : ∀(a:Abs).[](<State Int>(1 → a) → Int → a) @ .
```

Applying a modality to an absolute type

Modalities act only on non-absolute types, so a modality applied to an absolute type can always be discarded.

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Examples:

```
⊢ fun x → x : [Gen Int]List Int → List Int @ .
⊬ fun x → x : [Gen Int](1 → 1) → (1 → 1) @ .
a:Any ⊢ fun x → x : <State Int>([Gen Int]a) → [Gen Int]a @ .
a:Any ⊬ fun x → x : <State Int>a → a @ .
a:Abs ⊢ fun x → x : <State Int>a → a @ .
```

The kind restriction on effects

Operation arguments and results are restricted to be absolute.

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If we allowed `leak:(1 → 1) ↠ 1`, then we could write the following program

```
handle asList (fun () → do leak (fun () → do yield 42)) with
  return _  ⇒  fun () ⇒ 37
  leak p _  ⇒  p
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which leaks the `yield` operation

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which leaks the `yield` operation

Remark: it is possible to replace this restriction with an alternative formulation in which the order of higher-order effects is important.

Effect pollution

Read and fail effects

```
effect Read = ask:1 ↠ Int
effect Fail = fail:1 ↠ 0
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Handling reading from a list of integers (if the list is empty then reading fails):

```
reads : [Fail](<Read>(1 → Int) → List Int → Int)
reads f =
  handle f () with
    return v  ⇒  fun ns → v
    ask () r  ⇒  fun ns → case ns of
                    nil       ⇒  do fail ()
                    cons n ns ⇒  r n ns
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```

Handling failure as an option type:

```
maybeFail : [](<Fail>(1 → Int) → Maybe Int)
maybeFail f =
  handle f () with
    return v   ⇒  Just v
    fail () _  ⇒  Nothing
```

Effect pollution

Naively composing `reads` with `maybeFail` leaks the `Fail` effect:

```
bad : [](List Int → <Read, Fail>(1 → Int))
bad ns f = maybeFail (reads f ns)

bad [1,2] (fun () → (do ask ()) + (do fail ())) : Maybe Int @ .
```

This expression evaluates to `Nothing`.

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How can we **encapsulate** the use of `Fail` as an **intermediate** effect?

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How can we **encapsulate** the use of `Fail` as an **intermediate** effect?

The aim is to define

```
good : [](List Int → <Read>(1 → Int) → Maybe Int)
```

by composing `reads` and `maybeFail` such that

```
good [1,2] (fun () → (do ask ()) + (do fail ())) : Maybe Int @ Fail
```

performs the `fail` operation.

Effect encapsulation with masking

The solution is to **mask** the intermediate effect:

```
good : [](List Int → <Read>(1 → Int) → Maybe Int)
good ns f = maybeFail (reads (mask<fail> (f ())))
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The expression `mask<fail>(M)` masks `fail` from the ambient effect context for `M`.

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General form `<L|D>` specifies a transformation on effect contexts where:

- `L` is a row of effect labels that are removed from the effect context
- `D` is a row of effects that are added to the effect context

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General form `<L|D>` specifies a transformation on effect contexts where:

- `L` is a row of effect labels that are removed from the effect context
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`<D>` is shorthand for `<|D>`

Effect polymorphism

Higher-order cooperative concurrency effect

```
effect Coop = fork:[Coop](1 → 1) ↠ 1, suspend:1 ↠ 1
```

But the argument type of `fork` is absolute so cannot support other effects!

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```

Effect variables are **only needed** for use-cases such as higher-order effects where a computation must be stored for use in an effect context different from the ambient one.

In the paper (OOPSLA 2025)

Modal effect types — https://arxiv.org/abs/2407.11816

MET

- simply-typed multimodal core calculus with effects
- type system, operational semantics, type soundness, effect safety
- extensions: sums and products (crisp elimination), type and effect polymorphism

$\mathrm{F}^{1}_{\mathrm{eff}}$

- restricted core calculus of polymorphic effect types
- restriction: each scope can only refer to the lexically closest effect variables
- encoding of $\mathrm{F}^{1}_{\mathrm{eff}}$ in MET

METL: simple bidirectional type checking for MET

- infers all introduction and elimination of modalities
- analogous to generalisation and instantiation

In the follow-up paper (POPL 2026)

Rows and capabilities as modal effects (Wenhao Tang and Sam Lindley)

https://arxiv.org/abs/2507.10301

$\textsc{Met}(\mathcal{X})$

- abstracts over **effect structure** $\mathcal{X}$
- first class labels and modality parameterised handlers
- encodings of core calculi of Koka and Effekt in $\textsc{Met}(\mathcal{X})$

Ongoing and future work

Prototype implementations

Improved (bidirectional) type inference — Frost

Combination with Oxidizing OCaml (other modalities)

Denotational semantics

Inspirations

Do be do be do. *Lindley, McBride, and McLaughlin*. POPL 2017

Doo bee doo bee doo. *Convent, Lindley, McBride, and McLaughlin*. JFP 2020

Multimodal dependent type theory. *Gratzer, Kavvos, Nuyts, and Birkedal*. LMCS 2021

Effects, capabilities, and boxes: from scope-based reasoning to type-based reasoning and back. *Brachthäuser, Schuster, Lee, and Boruch-Gruszecki*. OOPSLA 2022

Oxidizing OCaml. *Lorenzen, White, Dolan, Eisenberg, and Lindley*. ICFP 2024