Modal effect types

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Effect polymorphism

A prototypical pure higher-order function

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We can only pass pure functions to `map`

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An effect polymorphic version

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No! In Frank the signature of `map` is syntactic sugar for the signature of `map'`.

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Key observation: almost always we need only one effect variable in a type signature

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For instance, consider a `yield` effect and a generator iterating over a list.

$$\texttt{gen} : \texttt{List Int} \xrightarrow{\texttt{yield}} \texttt{1}$$
$$\texttt{gen xs} = \texttt{map (fun x} \to \texttt{do yield x) xs; ()}$$

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gen : List Int -yield-> 1
gen xs = map (fun x → do yield x) xs; ()
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If we forget the annotation on the arrow then Frank gives the following error message.

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Can we do better?

From function arrows to effect contexts

Conventional effect typing — function arrows are annotated with effects

$\vdash\;\; \texttt{fun (f, x)} \rightarrow \texttt{f x} \;\;:\;\; ((\texttt{Int} \xrightarrow{\texttt{E}} \texttt{1}) \times \texttt{Int}) \xrightarrow{\texttt{E}} \texttt{1}$

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Modal effect typing — *ambient effect context* determines effects

$$\vdash \ \texttt{fun (}\underbrace{\texttt{f}}_{\texttt{@E}}\texttt{, x)} \ \underbrace{\to \texttt{f x}}_{\texttt{@E}} \ : \ ((\underbrace{\texttt{Int} \to \texttt{1}}_{\texttt{@E}}) \times \texttt{Int}) \ \underbrace{\to \texttt{1}}_{\texttt{@E}} \ \ \texttt{@} \ \ \texttt{E}$$

Effects contexts

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Effect context rows are *scoped* (as in Frank and Koka)

- repeats are allowed (same name but possibly different signatures)
- order of repeated operations matters
- relative order of distinct operations does not matter

Modal effect types

Modes are effect contexts
Modalities are transformations on modes

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$\textsc{Met}$ — simply-typed core calculus of modal effect types

$\textsc{Metl}$ — surface language for $\textsc{Met}$ with: bidirectional typing for inferring introduction and elimination of modalities + data types + polymorphism

Almost all examples in this talk use the simply-typed fragment of $\textsc{Metl}$

Overriding the ambient context with absolute modalities

$$\vdash \texttt{fun x} \to \underbrace{\texttt{do yield (x + 42)}}_{@\,\texttt{yield} : \texttt{Int} \twoheadrightarrow \texttt{1}} \; : \; [\texttt{yield} : \texttt{Int} \twoheadrightarrow \texttt{1}](\; \underbrace{\texttt{Int} \to \texttt{1}}_{@\,\texttt{yield} : \texttt{Int} \twoheadrightarrow \texttt{1}} \;) \; @ \;\; .$$

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The absolute modality $[\texttt{yield} : \texttt{Int} \twoheadrightarrow \texttt{1}]$ *overrides* the empty ambient effect context $(.)$ in the function body enabling the `yield` operation to be performed.

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Effect contexts given by absolute modalities percolate through the structure of a type:

- a function of type $\texttt{[E](A} \to \texttt{B)}$ may perform effects `E` when invoked
- elements of a list of type $\texttt{[E](List (A} \to \texttt{B))}$ may perform effects `E` when invoked

Effect context abbreviations

Example:

```
eff Gen a = yield : a ↠ 1
```

- `[Gen Int]` denotes the modality `[yield : Int ↠ 1]`
- `[Gen Int, E]` denotes the modality `[yield : Int ↠ 1, E]`

Absolute modalities and higher-order functions

Iteration specialised to integer lists:

```
iter : []((Int → 1) → List Int → 1)
iter f nil          = ()
iter f (cons x xs) = f x; iter f xs
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- *Boxing* = modality introduction
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In a conventional effect type system `iter` would be effect-polymorphic

$$\texttt{iter} : \forall\, \texttt{e} \;.\; (\texttt{Int} \xrightarrow{\texttt{e}} \texttt{1}) \xrightarrow{\texttt{e}} \texttt{List Int} \xrightarrow{\texttt{e}} \texttt{1}$$

Transforming the ambient context with relative modalities

Handling the `Gen Int` effect to produce a list of integers:

```
asList f =
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Unsound as it would allow us to write:

```
crash : [Gen String](String → List Int)
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`String` handled as `Int`!

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Sound, but consider:

$\vdash \texttt{fun}\ \underbrace{\texttt{f}}_{\texttt{@ Gen Int}}\ \to \texttt{handle}\ \underbrace{\texttt{f ()}}_{\texttt{@ Gen Int, E}}\ \texttt{with asList f : [Gen Int]}(\underbrace{\texttt{1} \to \texttt{1}}_{\texttt{@ Gen Int}}) \to \texttt{List Int @ E}$

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Restriction to `[Gen Int]` severely hinders resuability

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The *relative modality* `<Gen Int>` *extends* the ambient effect context.

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Now the effect context of `f` is `Gen Int, E`.

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Now the effect context of `f` is `Gen Int, E`.

In a conventional effect type system `asList` would be effect-polymorphic

$$\texttt{asList} : \forall\ \texttt{e}\ .\ (1 \xrightarrow{\texttt{Gen Int, e}} 1) \xrightarrow{\texttt{e}} \texttt{List Int}$$

Coercions between modalities

Automatic unboxing in METL allows values to be coerced between different modalities

We can extend an absolute modality:

$$\vdash \texttt{fun f} \to \texttt{f} : [\texttt{Gen Int}](\underbrace{\texttt{1} \to \texttt{1}}_{\texttt{@ Gen Int}}) \to [\texttt{Gen Int, Gen String}](\underbrace{\texttt{1} \to \texttt{1}}_{\texttt{@ Gen Int, Gen String}}) \ \texttt{@ E}$$

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We cannot extend a relative modality in the same way:

$$\nvdash \texttt{fun f} \to \texttt{f} : \texttt{<>}(\underbrace{1 \to 1}_{\texttt{@ E}}) \to \texttt{<Gen Int>}(\underbrace{1 \to 1}_{\texttt{@ Gen Int, E}})\ \texttt{@ E} \quad \texttt{\# Ill-typed}$$

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This would insert a fresh $\texttt{yield} : \texttt{Int} \twoheadrightarrow 1$ operation which may shadow other `yield` operations in `E`, permitting bad programs like `crash`.

Coercions between modalities

An absolute modality can be coerced into the corresponding relative modality.

$$\vdash \texttt{fun f} \rightarrow \texttt{f : [Gen Int]}(\underbrace{\texttt{1} \rightarrow \texttt{1}}_{\texttt{@ Gen Int}}) \rightarrow \texttt{<Gen Int>}(\ \underbrace{\texttt{1} \rightarrow \texttt{1}}_{\texttt{@ Gen Int, E}}\ ) \ \texttt{@ E}$$

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But the converse is not permitted

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as the argument may also use effects from the ambient effect context E.

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as the argument may also use effects from the ambient effect context `E`.

Similarly, the following typing judgement is invalid

$$\nvdash \texttt{fun}\ \underbrace{\texttt{f}}_{@\,\texttt{Gen Int, E}} \to \underbrace{\texttt{f ()}}_{@\,\texttt{E}} : \texttt{<Gen Int>}(\underbrace{1 \to 1}_{@\,\texttt{Gen Int, E}}) \to 1 \;@\; \texttt{E} \quad \texttt{\# Ill-typed}$$

as the argument may use `Gen Int` in addition to the ambient effect context `E`.

Composing handlers

State effect

```
eff State s = get : 1 ↠ s, put : s ↠ 1
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A state handler (specialised to integer state)

```
state : [](<State Int>(1 → 1) → Int → 1)
state m = handle m () with
  return x               ⇒ fun s → x
  get () r  ⇒ fun s → r s s
  put s' r  ⇒ fun s → r () s'
```

Composing handlers

Using integer state to write a generator that yields the prefix sum of a list

```
prefixSum : [Gen Int, State Int](List Int → 1)
prefixSum xs = iter (fun x → do put (do get () + x); do yield (do get ())) xs
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We can now handle the operations of `prefixSum` by composing two handlers

```
> asList (fun () → state (fun () → prefixSum [3,1,4,1,5,9]) 0)
# [3,4,8,9,14,23] : List Int
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In a conventional effect system composing handlers requires effect polymorphism

$$\texttt{asList} : \forall\, \texttt{e} \;.\; (1 \xrightarrow{\texttt{Gen Int, e}} 1) \xrightarrow{\texttt{e}} \texttt{List Int}$$

$$\texttt{state} : \forall\, \texttt{e} \;.\; (1 \xrightarrow{\texttt{State Int, e}} 1) \xrightarrow{\texttt{e}} \texttt{Int} \xrightarrow{\texttt{e}} 1$$

Storing effectful functions

First-order cooperative concurrency effect

```
eff Coop = suspend : 1 ↠ 1, ufork : 1 ↠ Bool
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Recursive data type of cooperative processes

```
data Proc = proc (List Proc → 1)

push : [](Proc → List Proc → List Proc)
push x xs = xs ++ cons x nil
```

```
next : [](List Proc → 1)
next q = case q of
  nil                  → ()
  cons (proc p) ps → p ps
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Scheduler parameterised by a list of suspended processes.

```
schedule : [](<Coop>(1 → 1) → List Proc → 1)
schedule m = handle m () with
  return ()              ⇒ fun q → next q
  suspend  () r ⇒ fun q → next (push (proc (r ())) q)
  ufork () r ⇒ fun q → r true (push (proc (r false)) q)
```

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In a conventional effect system storing effectful functions requires effect polymorphism

$$\texttt{data Proc e = proc (List Proc} \xrightarrow{\texttt{e}} \texttt{1)}$$

$$\texttt{schedule} : \forall\, \texttt{e} \,.\, (\texttt{1} \xrightarrow{\texttt{Coop, e}} \texttt{1}) \xrightarrow{\texttt{e}} \texttt{List (Proc e)} \xrightarrow{\texttt{e}} \texttt{1}$$

Masking

Using a generator to find an integer satisfying a predicate

```
findWrong : []((Int → Bool) → List Int → Maybe Int)  # ill-typed
findWrong p xs =
  handle (iter (fun x → if p x then do yield x) xs) with
    return _  ⇒ nothing
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Unsound to invoke `p` in the scope of the handler — would accidentally handle any yield operations performed by `p`

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$$\vdash \texttt{... handle (iter (fun x} \to \texttt{if } \underbrace{\texttt{(p x)}}_{\texttt{@ Gen Int, E}} \texttt{ then do yield x) xs) with ...} \quad : \quad \_ \quad @ \quad E$$

Changing the type of `p` to `<Gen Int>(Int → Bool)` fixes the type error but leaks the implementation detail that `findWrong` uses `yield`

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Unsound to invoke `p` in the scope of the handler — would accidentally handle any yield operations performed by `p`

$$\vdash \texttt{... handle (iter (fun x} \to \texttt{if } \underbrace{\texttt{(p x)}}_{\texttt{@ Gen Int, E}} \texttt{ then do yield x) xs) with ...} \quad : \quad \_ \quad @ \quad \texttt{E}$$

Changing the type of `p` to `<Gen Int>(Int → Bool)` fixes the type error but leaks the implementation detail that `findWrong` uses `yield`

Masking solves the problem

$$\vdash \texttt{... handle (iter (fun x} \to \texttt{if mask<yield>}\underbrace{\texttt{(p x)}}_{\texttt{@ E}} \texttt{ ...) with ...} \quad : \quad \_ \quad @ \quad \texttt{E}$$

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- `L` is a row of effect labels that are removed from the effect context
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`<D>` is shorthand for `<|D>`

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State handler for `1 → 1` computations

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state' (fun () → fun () → do put (do get () + 42)) 0  :  1 → 1
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- `state` cannot leak the state effect
- `state'` can leak the state effect

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- *Absolute types* formed only from base types and types boxed by an absolute modality cannot leak effects
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Subkinding allows absolute types to be treated as unrestricted

Value polymorphism

Polymorphic version of `iter`

```
iter : ∀ a . []((a → 1) → List a → 1)
iter {a} f nil          = ()
iter {a} f (cons x xs) = f x; iter {a} f xs
```

Explicit type abstractions and type applications in braces.

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Explicit type abstractions and type applications in braces.

Two possible polymorphic types for handling state

```
state  : ∀ [a] . [](<State Int>(1 → a) → Int → a)
state' : ∀  a  . [](<State Int>(1 → a) → Int → <State Int>a)
```

- ∀ `[a]` ascribes kind `Abs` to `a`, allowing values of type `a` to escape the handler.
- ∀ `a` ascribes kind `Any` to `a`, not allowing values of type `a` to escape the handler.

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- `∀ a` ascribes kind `Any` to `a`, not allowing values of type `a` to escape the handler.

Using η-expansion we can coerce `state'` to have the type of `state`

```
⊢ fun {a} m s → state' {a} m s : ∀ [a] . [](<State Int>(1 → a) → Int → a) @ .
```

The kind restriction on effects

Operation arguments and results are restricted to be absolute.

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If we allowed `leak : (1 → 1) ↠ 1`, then we could write the following program

```
handle asList (fun () → do leak (fun () → do yield 42)) with
  return                         _    ⇒ fun () ⇒ 37
  leak p _ ⇒ p
```

which leaks the `yield` operation

Effect polymorphism

Higher-order cooperative concurrency effect

$\texttt{eff Coop = fork : [Coop](1} \to \texttt{1)} \twoheadrightarrow \texttt{1, suspend : 1} \twoheadrightarrow \texttt{1}$

But the argument type of `fork` is absolute so cannot support other effects!

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Effect variables are *only needed* for use-cases such as higher-order effects where a computation must be stored for use in an effect context different from the ambient one.

In the paper

Modal effect types — https://arxiv.org/abs/2407.11816

$\textsc{Met}$

- simply-typed multimodal core calculus with effects
- type system, operational semantics, type soundness, effect safety
- extensions: sums and products (crisp elimination), value and effect polymorphism

$\mathrm{F}^1_{\mathrm{eff}}$

- restricted core calculus of polymorphic effect types
- restriction: each scope can only refer to the lexically closest effect variables
- encoding of $\mathrm{F}^1_{\mathrm{eff}}$ in $\textsc{Met}$

$\textsc{Metl}$: simple bidirectional type checking for $\textsc{Met}$

- infers all introduction and elimination of modalities
- analogous to generalisation and instantiation