Modal effect types

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Joint work with
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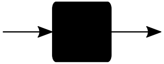
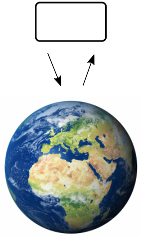
Effects

Programs as black boxes (Church-Turing model)?



Effects

Programs must interact with their environment



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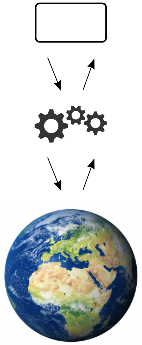

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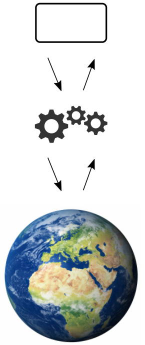

Effects are pervasive

- input/output
  user interaction
- concurrency
  web applications
- distribution
  cloud computing
- exceptions
  fault tolerance
- choice
  backtracking search

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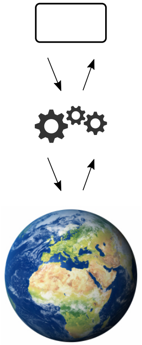

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  web applications
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Effect type systems statically track the use of effects

Conventional effect types

Pure computation

```
inc : Int → Int
inc i = i + 1

app : (Int → Int) → Int → Int
app f x = f x
```

```
> app inc 42
43 : Int
```

Conventional effect types

A variant of `inc` using a `Read` effect supporting effectful operation `ask : 1 ↠ Int`

```
inc : Int → Int
inc i = i + do ask ()
```

```
app : (Int → Int) → Int → Int
app f x = f x
```

Conventional effect types

Effects are tracked statically by adding effect annotations to arrows

```
inc : Int --Read--> Int
inc i = i + do ask ()

app : (Int → Int) → Int → Int
app f x = f x
```

Conventional effect types

Effect polymorphism allows `app` to be used in the presence of arbitrary effects

$$\texttt{inc} : \texttt{Int} \xrightarrow{\texttt{Read}} \texttt{Int}$$

```
inc i = i + do ask ()
```

$$\texttt{app} : \forall\, \texttt{e}.(\texttt{Int} \xrightarrow{\texttt{e}} \texttt{Int}) \xrightarrow{\texttt{e}} \texttt{Int} \xrightarrow{\texttt{e}} \texttt{Int}$$

```
app f x = f x
```

$$\texttt{appinc} : \texttt{Int} \xrightarrow{\texttt{Read}} \texttt{Int}$$

```
appinc = app inc
```

Conventional effect types

Effect polymorphism also allows `inc` to be used in contexts that have additional effects

$$\texttt{inc} : \forall\ \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read, e}} \texttt{Int}$$

```
inc i = i + do ask ()
```

$$\texttt{app} : \forall\ \texttt{e}.(\texttt{Int} \xrightarrow{\texttt{e}} \texttt{Int}) \xrightarrow{\texttt{e}} \texttt{Int} \xrightarrow{\texttt{e}} \texttt{Int}$$

```
app f x = f x
```

$$\texttt{inp} : \forall\ \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read, IO, e}} \texttt{Int}$$

```
inp i = do print "incrementing"; inc i
```

Conventional effect types

Effect polymorphism tracks a handler consuming an effect

$$\texttt{inc} : \forall\ \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read, e}} \texttt{Int}$$

```
inc i = i + do ask ()
```

$$\texttt{app} : \forall\ \texttt{e}.(\texttt{Int} \xrightarrow{\texttt{e}} \texttt{Int}) \xrightarrow{\texttt{e}} \texttt{Int} \xrightarrow{\texttt{e}} \texttt{Int}$$

```
app f x = f x
```

$$\texttt{two} : \forall\ \texttt{e}.(1 \xrightarrow{\texttt{Read, e}} \texttt{Int}) \xrightarrow{\texttt{e}} \texttt{Int}$$

```
two f = handle f () with {ask () r ⇒ r 2}
```

```
> two (fun () → app inc 42)
44 : Int
```

Conventional effect types

$$\texttt{inc} : \forall\ \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read, e}} \texttt{Int}$$

$$\texttt{inp} : \forall\ \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read, IO, e}} \texttt{Int}$$

$$\texttt{app} : \forall\ \texttt{e}.(\texttt{Int} \xrightarrow{\texttt{e}} \texttt{Int}) \xrightarrow{\texttt{e}} \texttt{Int} \xrightarrow{\texttt{e}} \texttt{Int}$$

$$\texttt{two} : \forall\ \texttt{e}.(\texttt{1} \xrightarrow{\texttt{Read, e}} \texttt{Int}) \xrightarrow{\texttt{e}} \texttt{Int}$$

Do we really need all of these effect variables?

Can we add effect types to existing languages without having to rewrite signatures of higher-order functions such `app`?

Modal effect types

Key ideas

- **decouple** effect types from function arrows
- track effects through an **ambient effect context**
- use **modalities** to modify the ambient effect context locally

Modal effect types

Pure computation

```
inc : Int → Int
inc i = i + 1

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```

```
> app inc 42
43 : Int
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Modal effect types

A variant of `inc` using a `Read` effect supporting effectful operation `ask : 1 ↠ Int`

```
inc : Int → Int
inc i = i + do ask ()
```

```
app : (Int → Int) → Int → Int
app f x = f x
```

Modal effect types

Effects (and purity) are tracked statically using **absolute modalities**

```
inc : [Read](Int → Int)
inc i = i + do ask ()

app : []((Int → Int) → Int → Int)
app f x = f x
```

Modal effect types

Subeffecting allows `app` to be used in the presence of arbitary effects

```
inc : [Read](Int → Int)
inc i = i + do ask ()

app : []((Int → Int) → Int → Int)
app f x = f x

appinc : [Read](Int → Int)
appinc = app inc
```

Modal effect types

Subeffecting also allows `inc` to be used in contexts that have other effects too

```
inc : [Read](Int → Int)
inc i = i + do ask ()

app : []((Int → Int) → Int → Int)
app f x = f x

inp : [Read, IO](Int → Int)
inp i = do print "incrementing"; inc i
```

Modal effect types

Relative modalities track a handler consuming an effect

```
inc : [Read](Int → Int)
inc i = i + do ask ()

app : []((Int → Int) → Int → Int)
app f x = f x

two : []((<Read>(1 → Int)) → Int)
two f = handle f () with {ask () r ⇒ r 2}

> two (fun () → app inc 42)
44 : Int
```

Comparing conventional effect types with modal effect types

Conventional effect types

$$\mathtt{inc} : \forall\ \mathtt{e}.\mathtt{Int} \xrightarrow{\mathtt{Read,\ e}} \mathtt{Int}$$

$$\mathtt{inp} : \forall\ \mathtt{e}.\mathtt{Int} \xrightarrow{\mathtt{Read,\ IO,\ e}} \mathtt{Int}$$

$$\mathtt{app} : \forall\ \mathtt{e}.(\mathtt{Int} \xrightarrow{\mathtt{e}} \mathtt{Int}) \xrightarrow{\mathtt{e}} \mathtt{Int} \xrightarrow{\mathtt{e}} \mathtt{Int}$$

$$\mathtt{two} : \forall\ \mathtt{e}.(\mathtt{1} \xrightarrow{\mathtt{Read,\ e}} \mathtt{Int}) \xrightarrow{\mathtt{e}} \mathtt{Int}$$

Modal effect types

$$\mathtt{inc} : [\mathtt{Read}](\mathtt{Int} \to \mathtt{Int})$$

$$\mathtt{inp} : [\mathtt{Read},\ \mathtt{IO}](\mathtt{Int} \to \mathtt{Int})$$

$$\mathtt{app} : [\,]((\mathtt{Int} \to \mathtt{Int}) \to \mathtt{Int} \to \mathtt{Int})$$

$$\mathtt{two} : [\,](\langle\mathtt{Read}\rangle(\mathtt{1} \to \mathtt{Int}) \to \mathtt{Int})$$

Comparing conventional effect types with modal effect types

Conventional effect types

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Modal effect types

$$\texttt{inc} : [\texttt{Read}](\texttt{Int} \rightarrow \texttt{Int})$$
$$\texttt{inp} : [\texttt{Read, IO}](\texttt{Int} \rightarrow \texttt{Int})$$
$$\texttt{app} : (\texttt{Int} \rightarrow \texttt{Int}) \rightarrow \texttt{Int} \rightarrow \texttt{Int}$$
$$\texttt{two} : \langle\texttt{Read}\rangle(\texttt{1} \rightarrow \texttt{Int}) \rightarrow \texttt{Int}$$

(we allow top-level empty absolute modalities to be omitted)

Comparing conventional effect types with modal effect types

Conventional effect types

$\texttt{inc} : \forall\ \texttt{e}.\texttt{Int} \xrightarrow{\texttt{Read, e}} \texttt{Int}$

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$\texttt{inc} : [\texttt{Read}](\texttt{Int} \to \texttt{Int})$

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Modal effect types allow us to avoid unnecessary effect polymorphism

From function arrows to effect contexts

Conventional effect typing — function arrows are annotated with effects

$$\vdash \ \texttt{fun (f, x)} \rightarrow \texttt{f x : ((Int} \xrightarrow{\texttt{E}} \texttt{1)} \times \texttt{Int)} \xrightarrow{\texttt{E}} \texttt{1}$$

From function arrows to effect contexts

Conventional effect typing — function arrows are annotated with effects

$$\vdash \ \texttt{fun (f, x)} \rightarrow \texttt{f x} : ((\texttt{Int} \xrightarrow{\texttt{E}} \texttt{1}) \times \texttt{Int}) \xrightarrow{\texttt{E}} \texttt{1}$$

Modal effect typing — **ambient effect context** determines effects

$$\vdash \ \texttt{fun (}\underbrace{\texttt{f}}_{\texttt{@ E}}\texttt{, x)} \underbrace{\rightarrow \texttt{f x}}_{\texttt{@ E}} : ((\underbrace{\texttt{Int} \rightarrow \texttt{1}}_{\texttt{@ E}}) \times \texttt{Int}) \underbrace{\rightarrow \texttt{1}}_{\texttt{@ E}} \ \ \texttt{@ E}$$

Effect contexts

An **effect context** `E` is a row of typed operations

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Example: `ask:1` $\twoheadrightarrow$ `Int`, `print:String` $\twoheadrightarrow$ `1`

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For convenience, we often group related operations together as effects

Examples:

```
eff Read = ask:1 ↠ Int
eff IO = print:String ↠ 1
eff State a = get:1 ↠ a, put:a ↠ 1
eff Gen a = yield:a ↠ 1
```

Effect contexts

An **effect context** `E` is a row of typed operations

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Examples:

```
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eff State a = get:1 ↠ a, put:a ↠ 1
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```

Effect context rows are **scoped** (as in Frank and Koka)

- repeats are allowed (same name but possibly different signatures)
- order of repeated operations matters
- relative order of distinct operations does not matter

Modal effect typing

A **mode** is an effect context

A **modality** is a transformation from one mode to another

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$\textsc{Met}$ — simply-typed core calculus of modal effect types

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METL — surface language for MET with: bidirectional typing for inferring introduction and elimination of modalities + algebraic data types + polymorphism

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METL — surface language for MET with: bidirectional typing for inferring introduction and elimination of modalities + algebraic data types + polymorphism

Almost all examples in this talk use the **simply-typed** fragment of METL

Overriding the ambient context with absolute modalities

$$\vdash \ \underbrace{\texttt{fun x} \to \texttt{do yield (x + 42)}}_{}\ $$

$$\vdash \ \texttt{fun x} \to \underbrace{\texttt{do yield (x + 42)}}_{\texttt{@ Gen Int}} \ \texttt{:} \ (\underbrace{\texttt{Int} \to \texttt{1}}_{\texttt{@ Gen Int}}) \ \texttt{@ Gen Int}$$

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The **absolute modality** `[Gen Int]` **overrides** the empty ambient effect context (.) in the function body enabling the `yield` operation to be performed.

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Effect contexts given by absolute modalities percolate through the structure of a type:

- a function of type `[E](A → B)` may perform effects `E` when invoked
- elements of a list of type `[E](List (A → B))` may perform effects `E` when invoked
- a value of type `[E]Int` cannot perform any effects

Absolute modalities and higher-order functions

Iteration specialised to integer lists:

```
iter : []((Int → 1) → List Int → 1)
iter f nil          = ()
iter f (cons x xs) = f x; iter f xs
```

Absolute modalities and higher-order functions

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Applying a pure higher-order function in an impure effect context:

```
⊢  iter (fun x → do yield (x + 42)) : 1 @ Gen Int
```

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- **boxing** = modality introduction
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Terminology:

- **boxing** = modality introduction
- **unboxing** = modality elimination

In a conventional effect type system `iter` would be effect-polymorphic

$$\texttt{iter} : \forall\, \texttt{e}.(\texttt{Int} \xrightarrow{\texttt{e}} \texttt{1}) \xrightarrow{\texttt{e}} \texttt{List Int} \xrightarrow{\texttt{e}} \texttt{1}$$

Transforming the ambient context with relative modalities

Handling the `Gen Int` effect to produce a list of integers:

```
asList : [](<Gen Int>(1 → 1) → List Int)
asList f =
  handle f () with
    return ()  ⇒  nil
    yield x r  ⇒  cons x (r ())
```

Transforming the ambient context with relative modalities

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The **relative modality** `<Gen Int>` **extends** the ambient effect context.

$$\vdash \texttt{fun}\ \underbrace{\texttt{f}}_{\texttt{@ Gen Int, E}} \rightarrow \texttt{handle}\ \underbrace{\texttt{f ()}}_{\texttt{@ Gen Int, E}}\ \texttt{with} \ldots : \texttt{<Gen Int>(}\ \underbrace{1 \rightarrow 1}_{\texttt{@ Gen Int, E}}\ \texttt{)} \rightarrow \texttt{List Int @ E}$$

The effect context of `f` is `Gen Int, E`.

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The effect context of `f` is `Gen Int, E`.

In a conventional effect type system `asList` would be effect-polymorphic

$$\texttt{asList} : \forall\ \texttt{e}.(1 \xrightarrow{\texttt{Gen Int, e}} 1) \xrightarrow{\texttt{e}} \texttt{List Int}$$

A flavour of the typing rules (courtesy of Wenhao Tang)

A Tale of Locks 🔒 and Keys 🔑

mod introduces a modality and a lock

the ambient effect context is overwritten to ask

$$\frac{\Gamma, \text{🔒}_{[ask]} \vdash \mathbf{fun}\ x \to f\ x : \mathtt{Int} \to \mathtt{Int}\ @\ ask}{\Gamma \vdash \mathbf{mod}_{[ask]}\ (\mathbf{fun}\ x \to f\ x) : [ask](\mathtt{Int} \to \mathtt{Int})\ @\ E}$$

locks control usage of variables

modality transformation: the key 🔑 to the lock

$$\frac{[\,] \Rightarrow [ask]}{f :_{[\,]} \mathtt{Int} \to \mathtt{Int}, \text{🔒}_{[ask]} \vdash f : \mathtt{Int} \to \mathtt{Int}\ @\ ask}$$

$$\frac{\Gamma \vdash V : [\,](\mathtt{Int} \to \mathtt{Int})\ @\ E \qquad \Gamma, f :_{[\,]} \mathtt{Int} \to \mathtt{Int} \vdash M : A\ @\ E}{\Gamma \vdash \mathbf{let}\ \mathbf{mod}_{[\,]}\ f = V\ \mathbf{in}\ M : A\ @\ E}$$

let mod eliminates a modality and introduces a binder with a modality

Coercions between modalities

Automatic unboxing in METL allows values to be coerced between different modalities

We can extend an absolute modality:

$\vdash$ `fun f` $\rightarrow$ `f : [Gen Int](1` $\rightarrow$ `1)` $\rightarrow$ `[Gen Int, Gen String](1` $\rightarrow$ `1) @ E`

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$\vdash$ `fun f -> f : [Gen Int](1 -> 1) -> [Gen Int, Gen String](1 -> 1) @ E`

In a conventional effect type system this corresponds to:

$$\vdash \texttt{fun f} \to \texttt{f} : (\forall\ \texttt{e}.\texttt{1} \xrightarrow{\texttt{Gen Int, e}} \texttt{1}) \xrightarrow{\texttt{E}} (\forall\ \texttt{e}.\texttt{1} \xrightarrow{\texttt{Gen Int, Gen String, e}} \texttt{1})$$

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We cannot extend a relative modality in the same way:

$\nvdash$ `fun f → f : <>(1 → 1) → <Gen Int>(1 → 1) @ E  # Ill-typed`

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This would insert a fresh `yield:Int` $\twoheadrightarrow$ `1` operation which may shadow other `yield` operations in `E`.

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This would insert a fresh `yield:Int ->> 1` operation which may shadow other `yield` operations in `E`.

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An absolute modality can be coerced into the corresponding relative modality.

$\vdash$ `fun f → f : [Gen Int](1 → 1) → <Gen Int>(1 → 1) @ E`

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$\vdash$ `fun f -> f : [Gen Int](1 -> 1) -> <Gen Int>(1 -> 1) @ E`

In a conventional effect type system this corresponds to:

$\vdash \texttt{fun f} \to \texttt{f : } (\forall\ \texttt{e}.\texttt{1} \xrightarrow{\texttt{Gen Int, e}} \texttt{1}) \xrightarrow{\texttt{E}} (\texttt{1} \xrightarrow{\texttt{Gen Int, E}} \texttt{1})$

But the converse is not permitted

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Composing handlers

State effect

```
eff State s = get:1 ↠ s, put:s ↠ 1
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eff State s = get:1 ↠ s, put:s ↠ 1
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A state handler (specialised to integer state)

```
state : [](<State Int>(1 → 1) → Int → 1)
state m = handle m () with
  return x  ⇒  fun s → x
  get () r  ⇒  fun s → r s s
  put s' r  ⇒  fun s → r () s'
```

Composing handlers

Using integer state to write a generator that yields the prefix sum of a list

```
prefixSum : [Gen Int, State Int](List Int → 1)
prefixSum xs = iter (fun x → do put (do get () + x); do yield (do get ())) xs
```

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We can now handle the operations of `prefixSum` by composing two handlers

```
> asList (fun () → state (fun () → prefixSum [3,1,4,1,5,9]) 0)
# [3,4,8,9,14,23] : List Int
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# [3,4,8,9,14,23] : List Int
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In a conventional effect system composing handlers requires effect polymorphism

$$\texttt{asList} : \forall\, \texttt{e}.(1 \xrightarrow{\texttt{Gen Int, e}} 1) \xrightarrow{\texttt{e}} \texttt{List Int}$$

$$\texttt{state} : \forall\, \texttt{e}.(1 \xrightarrow{\texttt{State Int, e}} 1) \xrightarrow{\texttt{e}} \texttt{Int} \xrightarrow{\texttt{e}} 1$$

Storing effectful functions

First-order cooperative concurrency effect

```
eff Coop = suspend:1 ↠ 1, ufork:1 ↠ Bool
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Recursive data type of cooperative processes

```
data Proc = proc (List Proc → 1)

push : [](Proc → List Proc → List Proc)
push x xs = xs ++ cons x nil
```

```
next : [](List Proc → 1)
next q = case q of
  nil                → ()
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Scheduler parameterised by a list of suspended processes

```
schedule : [](<Coop>(1 → 1) → List Proc → 1)
schedule m = handle m () with
  return ()      ⇒  fun q → next q
  suspend () r   ⇒  fun q → next (push (proc (r ())) q)
  ufork () r     ⇒  fun q → r true (push (proc (r false)) q)
```

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In a conventional effect system storing effectful functions requires effect polymorphism

$$\texttt{data Proc e = proc (List Proc} \xrightarrow{\texttt{e}} \texttt{1)}$$

$$\texttt{schedule : } \forall\, \texttt{e.(1} \xrightarrow{\texttt{Coop, e}} \texttt{1)} \xrightarrow{\texttt{e}} \texttt{List (Proc e)} \xrightarrow{\texttt{e}} \texttt{1}$$

Kinds

State handler for `1` $\to$ `1` computations

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```

- `state` cannot leak the state effect
- `state'` can leak the state effect

Kinds

- **Absolute types** (e.g. 1, List Int, and [Gen Int](List Int → 1))
  built from base types, positive types, and types boxed by an absolute modality —
  cannot leak effects
- **Unrestricted types** (e.g. 1 → 1, List Int → 1, and <Coop>(1 → 1))
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- Any classifies unrestricted types

Subkinding allows absolute types to be treated as unrestricted: Abs $\leq$ Any

Type polymorphism

Polymorphic version of `iter`

```
iter : ∀(a:Any).[]((a → 1) → List a → 1)
iter {a:Any} f nil          = ()
iter {a:Any} f (cons x xs) = f x; iter {a} f xs
```

Explicit type abstractions and type applications in braces.

Type polymorphism

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Explicit type abstractions and type applications in braces.

Two possible polymorphic types for handling state

```
state  : ∀(a:Abs).[](<State Int>(1 → a) → Int → a)
state' : ∀(a:Any).[](<State Int>(1 → a) → Int → <State Int>a)
```

- `∀(a:Abs)` ascribes kind `Abs` to `a`, allowing values of type `a` to escape the handler.
- `∀(a:Any)` ascribes kind `Any` to `a`, not allowing values of type `a` to escape the handler.

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- `∀(a:Any)` ascribes kind `Any` to `a`, not allowing values of type `a` to escape the handler.

Using η-expansion we can coerce `state'` to have the type of `state`

```
⊢fun {a:Abs} m s → state' {a} m s : ∀(a:Abs).[](<State Int>(1 → a) → Int → a) @ .
```

Applying a modality to an absolute type

Modalities act only on non-absolute types, so a modality applied to an absolute type can always be discarded.

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Examples:

```
⊢ fun x → x : [Gen Int]List Int → List Int @ .
⊬ fun x → x : [Gen Int](1 → 1) → (1 → 1) @ .
a:Any ⊢ fun x → x : <State Int>([Gen Int]a) → [Gen Int]a @ .
a:Any ⊬ fun x → x : <State Int>a → a @ .
a:Abs ⊢ fun x → x : <State Int>a → a @ .
```

The kind restriction on effects

Operation arguments and results are restricted to be absolute.

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If we allowed `leak:(1 → 1) ↠ 1`, then we could write the following program

```
handle asList (fun () → do leak (fun () → do yield 42)) with
  return _  ⇒  fun () ⇒ 37
  leak p _  ⇒  p
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which leaks the `yield` operation

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which leaks the `yield` operation

Remark: it is possible to replace this restriction with an alternative formulation in which the order of higher-order effects is important.

Effect pollution

Read and fail effects

```
eff Read = ask : 1 → Int
eff Fail = fail : 1 → 0
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Handling reading from a list of integers (if the list is empty then reading fails):

```
reads : [Fail](<Read>(1 → Int) → List Int → Int)
reads f =
  handle f () with
    return v  ⇒  fun ns → v
    ask () r  ⇒  fun ns → case ns of
                    nil        ⇒  do fail ()
                    cons n ns  ⇒  r n ns
```

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```

Handling failure as an option type:

```
maybeFail : [](<Fail>(1 → Int) → Maybe Int)
maybeFail f =
  handle f () with
    return v   ⇒  Just v
    fail () _  ⇒  Nothing
```

Effect pollution

Naively composing `reads` with `maybeFail` leaks the `Fail` effect:

```
bad : [](List Int → <Read, Fail>(1 → Int))
bad ns f = maybeFail (reads f ns)

bad [1,2] (fun () → (do ask ()) + (do fail ())) : Maybe Int @ .
```

This expression evaluates to `Nothing`.

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How can we **encapsulate** the use of `Fail` as an **intermediate** effect?

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This expression evaluates to `Nothing`.

How can we **encapsulate** the use of `Fail` as an **intermediate** effect?

The aim is to define

```
good : [](List Int → <Read>(1 → Int) → Maybe Int)
```

by composing `reads` and `maybeFail` such that

```
good [1,2] (fun () → (do ask ()) + (do fail ())) : Maybe Int @ Fail
```

performs the `fail` operation.

Effect encapsulation with masking

The solution is to **mask** the intermediate effect:

```
good : [](List Int → <Read>(1 → Int) → Maybe Int)
good ns f = maybeFail (reads (mask<fail> (f ())))
```

The expression `mask<fail>(M)` masks `fail` from the ambient effect context for `M`.

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General form <L|D> specifies a transformation on effect contexts where:

- L is a row of effect labels that are removed from the effect context
- D is a row of effects that are added to the effect context

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General form `<L|D>` specifies a transformation on effect contexts where:

- `L` is a row of effect labels that are removed from the effect context
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`<D>` is shorthand for `<|D>`

Effect polymorphism

Higher-order cooperative concurrency effect

```
eff Coop = fork:[Coop](1 → 1) ↠ 1, suspend:1 ↠ 1
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But the argument type of `fork` is absolute so cannot support other effects!

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METL includes effect polymorphism to support higher-order operations like `fork`

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METL includes effect polymorphism to support higher-order operations like `fork`

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eff Coop e = fork:[Coop e, e](1 → 1) ↠ 1, suspend:1 ↠ 1
```

Effect variables are **only needed** for use-cases such as higher-order effects where a computation must be stored for use in an effect context different from the ambient one.

In the paper (OOPSLA 2025)

Modal effect types — https://arxiv.org/abs/2407.11816

$\textsc{Met}$

- simply-typed multimodal core calculus with effects
- type system, operational semantics, type soundness, effect safety
- extensions: sums and products (crisp elimination), type and effect polymorphism

$\mathrm{F}^1_{\mathrm{eff}}$

- restricted core calculus of polymorphic effect types
- restriction: each scope can only refer to the lexically closest effect variables
- encoding of $\mathrm{F}^1_{\mathrm{eff}}$ in $\textsc{Met}$

$\textsc{Metl}$: simple bidirectional type checking for $\textsc{Met}$

- infers all introduction and elimination of modalities
- analogous to generalisation and instantiation

In the follow-up paper (POPL 2026)

Rows and capabilities as modal effects (Wenhao Tang and Sam Lindley)

https://arxiv.org/abs/2507.10301

$\textsc{Met}(\mathcal{X})$

- abstracts over **effect structure** $\mathcal{X}$
- first class labels and modality parameterised handlers
- encodings of core calculi of Koka and Effekt in $\textsc{Met}(\mathcal{X})$

Ongoing and future work

Denotational semantics

Prototype implementation of METL

Improved (bidirectional) type inference — Frost

Combination with oxidizing OCaml (other modalities)

Inspirations

Do be do be do. *Lindley, McBride, and McLaughlin*. POPL 2017

Doo bee doo bee doo. *Convent, Lindley, McBride, and McLaughlin*. JFP 2020

Effekt. *Brachthäuser, Schuster, and Ostermann*

Oxidizing OCaml. *Lorenzen, White, Dolan, Eisenberg, and Lindley*. ICFP 2024

Multimodal dependent type theory. *Gratzer, Kavvos, Nuyts, and Birkedal*. LMCS 2021