Paella: algebraic effects with parameters and their handlers

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Overview

Overview

Algebraic effects and handlers

- Why?
  - **User-defined** computational effects
  - Mathematically structured
- Examples
  - Backtracking choice
  - Global state
  - Exceptions
  - Yielding
- Implementation
  - Programs represent **computation trees**
  - Handlers **fold** over these trees

State effects

- Effects for static state:

```
read : Loc -> Bit
write : (Loc, Bit) -> ()
```

- Effects for dynamically-allocated state:

```
new : Bit -> Loc
gc : Policy -> ()
```

Problems for state effects

- To support `new` and `gc`, `Loc` needs to be "abstract" and/or "dynamic"
  - Avoid counterfeit locations
  - Change when memory cell moves
- E.g. capturing a reference in a closure

```
ExDangling = do
  loc <- new I
  let kont = \_ => write (loc, 0) -- Capture `loc` in closure
  _ <- gc Compact
  kont () -- Writing to possibly dangling pointer!
```

Overview

Algebraic effects and handlers

Ordinary computation trees

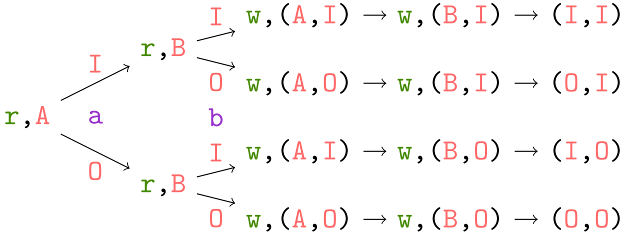
An example program

Kripke computation trees

Conclusion

Algebraic effects and handlers: computation trees

```
ExGS : (OpGS).Free (Bit, Bit)
ExGS = do -- Swaps values
  a <- read A
  b <- read B
  () <- write (A, b)
  () <- write (B, a)
  pure (b, a) -- (A,B)
```




```
data Loc = A | B
data Bit = O | I
```

Algebraic effects and handlers: computation trees

```
ExGS : (OpGS).Free (Bit, Bit)
ExGS = do -- Swaps values
  a <- read A
  b <- read B
  () <- write (A
  () <- write (B
  pure (b, a) -- (A,B)
```

```
I → r,B → I w,(A,I) → w,(B,I) → (I,I)
→ w,(B,I) → (O,I)
→ w,(B,O) → (I,O)
O w,(A,O) → w,(B,O) → (O,O)
```

```
data OpGS : AlgSignature where
  Read  : OpGS (Loc ~|> Bit)
  Write : OpGS ((Loc, Bit) ~|> ())
```

```
data Loc = A | B
data Bit = O | I
```

Algebraic effects and handlers: core types

```
record AlgOpSig where
  constructor (~|>)
  Args, Arity : Type

AlgSignature : Type
AlgSignature = AlgOpSig -> Type

data (.Free) : AlgSignature -> Type -> Type where
  Return : x -> sig.Free x
  Op : sig opSig ->
    (opSig.Args, opSig.Arity -> sig.Free x) -> sig.Free x
```

Algebraic effects and handlers: core types

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record AlgOpSig where
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```

Algebraic effects and handlers: core types with parameters

```
record AlgOpSig where
  constructor (~|>)
  Args, Arity : Family

AlgSignature : Type
AlgSignature = AlgOpSig -> Type

data (.Free) : AlgSignature -> Family -> Family where
  Return : x -|> sig.Free x
  Op : sig opSig ->
    FamProd [< opSig.Args, opSig.Arity -% sig.Free x] -|> sig.Free x
```

Overview

Ticking: operations

Two kinds of dynamic values (Var): ticking Clocks and cooperative Tasks (cf. [Hillerström and KC(2017), Matache(2024)]).

```
data TickySig : Cell .signature where
  /// Allocate a fresh clock initialised by argument
  New  : TickySig (const Nat ~|> Var Clock)
  /// Synchronise two clocks,adding their ticks
  Sync : TickySig (FamProd [< Var Clock, Var Clock] ~|> const ())
  /// Tick a clock
  Work : TickySig (FamProd [< Var Clock, const Nat] ~|> const ())
  /// Wait until clock ticks past argument
  WaitUntil : TickySig (FamProd [< Var Clock, const Nat] ~|> const ())
  /// Threading a-la Unix interface [cf. Matache'24]
  Stop : TickySig (const () ~|> const Void)
  Fork : TickySig (const () ~|> FamSum [< Var Task, const ()])
  Wait : TickySig (Var Task ~|> const ())
```

Ticking: example program

- Consider the example program:

```
ExTicks =                    (\w, [<                ] =>
  new  _ 0            ) >>== (\w, [< c1             ] =>
  new  _ 1            ) >>== (\w, [< c1, c2         ] =>
  new  _ 3            ) >>== (\w, [< c1, c2, c3     ] =>
  work _ [< c1, 50])  >>>>   (\w, [< c1, c2, c3     ] =>
  work _ [< c2, 60])  >>>>   (
  forkOff             $      (\w, [< c1, c2, c3     ] =>
    waitUntil
       _ [< c3, 40])  >>>>   (\w, [< c1, c2, c3     ] =>
    work
       _ [< c1, 10])
  )                   >>==   (\w, [< c1, c2, c3, tid] =>
  sync _ [< c1, c3])  >>>>   (\w, [< c1, c2, c3, tid] =>
  wait _ tid          ) >>>> (\w, [< c1, c2, c3, tid] =>
  sync _ [< c1, c2])
```

Ticking: example program

- Consider the example program:

```
ExTicks =                      (\w, [<                    ] =>
  new  _ 0           ) >>== (\w, [< c1                ] =>
  new  _ 1           ) >>== (\w, [< c1, c2            ] =>
  new  _ 3           ) >>== (\w, [< c1, c2, c3        ] =>
  work _ [< c1, 50]) >>>> (\w, [< c1, c2,
  work _ [< c2, 60]) >>>> (
  forkOff            $      (\w, [< c1, c2,
    waitUntil
      _ [< c3, 40]) >>>> (\w, [< c1, c2,
    work
      _ [< c1, 10])
  )                    >>== (\w, [< c1, c2, c3, tid] =>
  sync _ [< c1, c3]) >>>> (\w, [< c1, c2, c3, tid] =>
  wait _ tid         ) >>>> (\w, [< c1, c2, c3, tid] =>
  sync _ [< c1, c2])
```

$$\frac{\Gamma \vdash t : MA \qquad \Gamma, x : A \vdash u : MB}{\Gamma \vdash \text{let } x = t \text{ in } u : MB}$$

Ticking: example program

- Consider the example program:

```
ExTicks =                    (\w, [<                ] =>
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       _ [< c1, 10])
  )                  >>== (\w, [< c1, c2, c3, tid] =>
  sync _ [< c1, c3]) >>>> (\w, [< c1, c2, c3, tid] =>
  wait _ tid       ) >>>> (\w, [< c1, c2, c3, tid] =>
  sync _ [< c1, c2])
```

Ticking: example program

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  work _ [< c1, 50]) >>>> (\w, [< c1
  work _ [< c2, 60]) >>>> (
  forkOff            $      (\w, [< c1, c2, c3     ] =>
    waitUntil
      _ [< c3, 40]) >>>> (\w, [< c1, c2, c3     ] =>
    work
      _ [< c1, 10])
  )                    >>== (\w, [< c1, c2, c3, tid] =>
  sync _ [< c1, c3]) >>>> (\w, [< c1, c2, c3, tid] =>
  wait _ tid          ) >>>> (\w, [< c1, c2, c3, tid] =>
  sync _ [< c1, c2])
```

Time aware continuations; when work ticks and sync merges clocks, the rest of the program is updated

Ticking: takeaways

- `Var Clock` has dynamic values which change based on the **world**, in this which clocks and tasks are at large
- Each `new` and `fork` change the world
- Can't be solved by naive IO references due to clock synchronisation (requires another level of indirection)
- `Clock` and `Task` are example **parameters** [Staton(2013)]
- Parameter for local state: the shape of the heap
- Enter **Paella**, a **pa**rameterised **al**gebraic **e**ffects **l**ibrary/**la**nguage

Idea: do the same, but with world-aware types, i.e. Kripke semantics

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Kripke semantics: worlds

```
World : Type

infixr 1 ~>
(~>) : (src, tgt : World) -> Type

id : t ~> t

infixr 9 .
(.) : (t2 ~> t3) -> (t1 ~> t2) -> (t1 ~> t3)
```

Kripke semantics: families

```
Family : Type
Family = World -> Type

infixr 1 -|>
(-|>) : (f, g : Family) -> Type
f -|> g = (w : World) -> f w -> g w

id : {f : Family} -> f -|> f
id w x = x

infixr 9 .
(.) : {f, g, h : Family} -> (g -|> h) -> (f -|> g) -> (f -|> h)
(beta . alpha) w = beta w . alpha w
```

Kripke semantics: signatures and computation trees

```
record OpSig where
  constructor (~|>)
  Args   : Family
  Arity : Family

Signature : Type
Signature = OpSig -> Type

data (.Free) : Signature -> Family -> Family where
  Return : f -|> sig.Free f -- (w : World) -> f w -> sig.Free f w
  Op : {opSig : OpSig} -> {f : Family} -> (op : sig opSig) ->
    FamProd [< opSig.Args, opSig.Arity -% sig.Free f] -|> sig.Free f
```

Kripke semantics: families with actions (presheaves)

```
ActionOver : Family -> Type
ActionOver f = {w, w' : World} -> (rho : w ~> w') -> (f w -> f w')

Box : Family -> Family
Box f w = (w' : World) -> (w ~> w') -> f w'

record BoxCoalg (f : Family) where
  constructor MkBoxCoalg
  next : f -|> Box f
  -- (w : World) -> f w -> (w' : World) -> (w ~> w') -> f w'

(.map) : {f : Family} -> BoxCoalg f -> ActionOver f
coalg.map {w,w'} rho x = coalg.next w x w' rho
```

See [Allais et al.(2018), Fiore and Szamozvancev(2022)] for this approach

Kripke semantics: basic families with actions

```
BoxCoalgConst : {t : Type} -> BoxCoalg (const t)
BoxCoalgConst = MkBoxCoalg $ \_, x, _, _ => x

Env : World -> Family
Env w = (w ~>)

BoxCoalgEnv : {w0 : World} -> BoxCoalg (Env w0)
BoxCoalgEnv = MkBoxCoalg $ \w, rho, w', rho' => rho' . rho
-- rho  : w0 ~> w
-- rho' : w  ~> w'
```

Kripke semantics: product of families with actions

```
data ForAll : SnocList a -> (a -> Type) -> Type where
  Lin  : ForAll sx p
  (:<) : ForAll sx p -> p x -> ForAll (sx :< x) p

FamProd : SnocList Family -> Family
FamProd sf w = ForAll sf (\f => f w)

BoxCoalgProd : {sf : SnocList Family} ->
  ForAll sf BoxCoalg -> BoxCoalg $ FamProd sf
```

Kripke semantics: exponential of families with actions

```
(-%) : (f, g : Family) -> Family
(f -% g) w = (FamProd [< Env w, f]) -|> g
-- (w' : World) -> (w ~> w') -> f w' -> g w'

eval : FamProd [< f -% g, f] -|> g
eval w [< alpha, x] = alpha w [< id, x]

(.curry) : {h : Family} -> (coalg : BoxCoalg h) ->
  (FamProd [< h, f] -|> g) -> (h -|> (f -% g))

BoxCoalgExp : BoxCoalg (f -% g)
BoxCoalgExp = MkBoxCoalg $ \w, alpha, w', rho =>
  \w'', [< rho', x] => alpha w'' [< rho' . rho, x]
-- rho  : w  ~> w'
-- rho' : w' ~> w''
```

Kripke semantics: signatures and computation trees again

```
record OpSig where
  constructor (~|>)
  Args, Arity  : Family

Signature : Type
Signature = OpSig -> Type

data (.Free) : Signature -> Family -> Family where
  Return : f -|> sig.Free f
  Op : {opSig : OpSig} -> {f : Family} -> (op : sig opSig) ->
    FamProd [< opSig.Args, opSig.Arity -% sig.Free f] -|> sig.Free f
```

Kripke semantics: signatures and computation trees again

```
recor
   con
   Arg

Signa
Signa
```

Inlining the abstractions:

```
(w : World) ->
  ( opSig.Args w,
    (w' : World) -> (w ~> w') -> opSig.Arity w' -> sig.Free f w'
  )
-> sig.Free f w
```

```
data (.Free) : Signature -> Family -> Family where
  Return : f -|> sig.Free f
  Op : {opSig : OpSig} -> {f : Family} -> (op : sig opSig) ->
    FamProd [< opSig.Args, opSig.Arity -% sig.Free f] -|> sig.Free f
```

Overview

Recap of Kripke semantics

- We defined a type of worlds and families over such worlds
- We defined families with actions (presheaves), as well as their products and exponentials
- We defined new computation trees with branching that supports any future world
- These trees have an action and a folding operation
- They also form a monad, and so we can create computations which are updatable!

Prospects

- Applications
  - Full ground local state (i.e. ground values and pointers) and the Tarjan-Sleator transform (WIP in Idris 2)
  - Elaboration and constraint solving with meta-variables (already in Haskell)
  - Threads (WIP in Idris 2)
- Improved ergonomics
  - Semantic reflection for Idris 2
  - Type classes and local instances (already in Haskell)

Repository: `https://github.com/ohad/paella`

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