Algebraic effects and effect handlers

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FoPSS 2026

Quiz

Quiz

What is an effect?

Quiz

What is an effect?

What is a pure computation?

Quiz

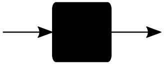
What is an effect?

What is a pure computation?

What is an effectful computation?

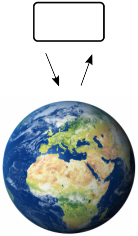
Effects

Programs as black boxes (Church-Turing model)?



Effects

Programs must interact with their environment



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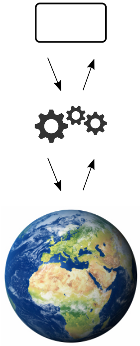

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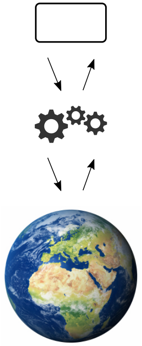
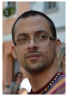


Effects are pervasive

- input/output
  user interaction
- concurrency
  web applications
- distribution
  cloud computing
- exceptions
  fault tolerance
- choice
  backtracking search

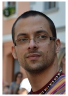
Typically ad hoc and hard-wired

Effect handlers

 Gordon Plotkin  Matija Pretnar

Handlers of algebraic effects, ESOP 2009 (and ETAPS 2022 test of time award)

Effect handlers

 Gordon Plotkin  Matija Pretnar

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Monads $\longrightarrow$ Algebraic Effects $\longrightarrow$ Effect Handlers

Effect handlers

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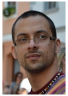

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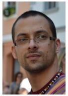
Composable and **customisable** user-defined interpretation of effects in general

Effect handlers

 Gordon Plotkin  Matija Pretnar

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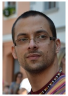
Give programmer direct access to **context**
(c.f. resumable exceptions, monads, delimited control)

Effect handlers

 Gordon Plotkin  Matija Pretnar

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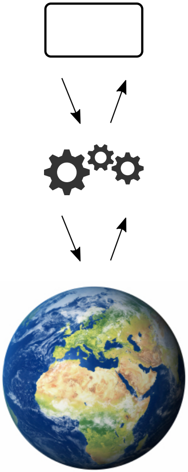
Monads $\longrightarrow$ Algebraic Effects $\longrightarrow$ Effect Handlers

Composable and **customisable** user-defined interpretation of effects in general

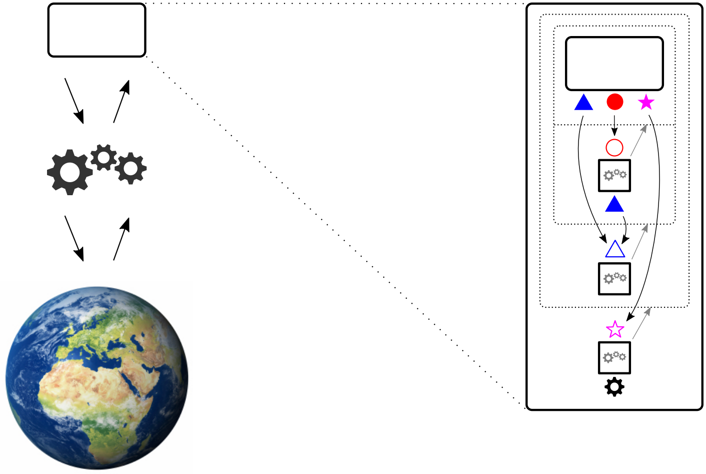
Give programmer direct access to **context**
(c.f. resumable exceptions, monads, delimited control)

Effect handlers in practice:
OCaml 5, GitHub (Semantic), Meta (React), Uber (Pyro), WasmFX, ...

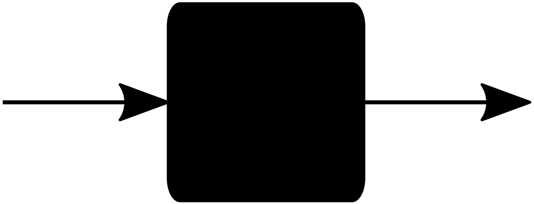
Effect handlers as composable user-defined operating systems



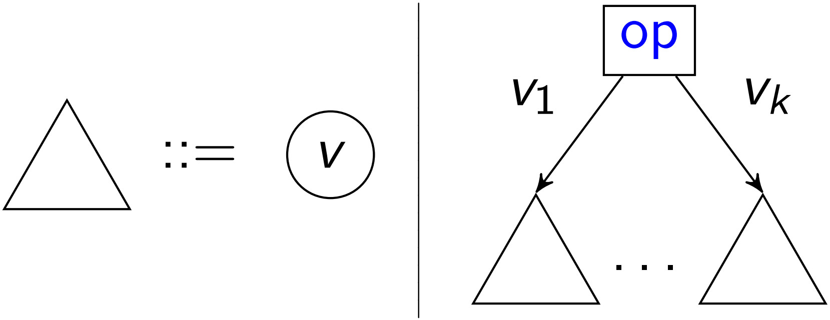
Effect handlers as composable user-defined operating systems



Pure computations

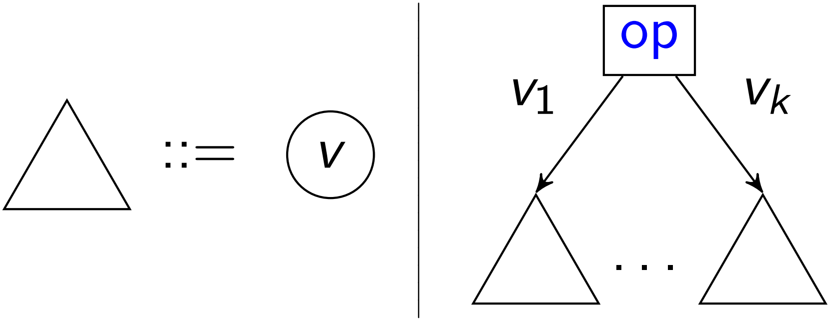


Effectful computations



A **command-response** tree (aka **interaction tree**)

Effectful computations

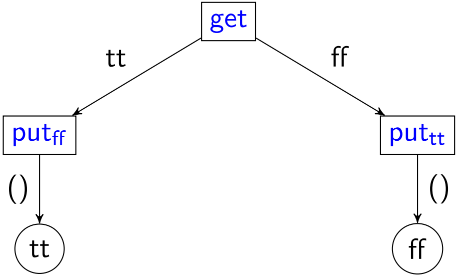


A **command-response** tree (aka **interaction tree**)

Effectful computation is all about **interaction** with some **context**

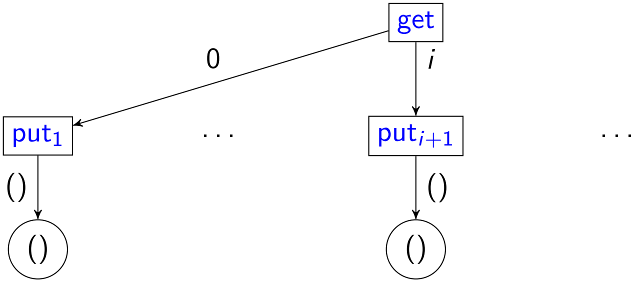
Example: boolean state (bit toggling)

$$\begin{aligned} \mathsf{get} &: \mathsf{Bool} \\ \mathsf{put}_{\mathsf{tt}} &: 1 \\ \mathsf{put}_{\mathsf{ff}} &: 1 \end{aligned}$$



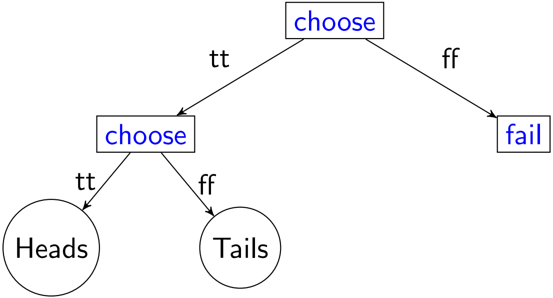
Example: natural number state (increment)

$$\begin{aligned} \mathsf{get} &: \mathbb{N} \\ \mathsf{put}_i &: 1, \quad i \in \mathbb{N} \end{aligned}$$

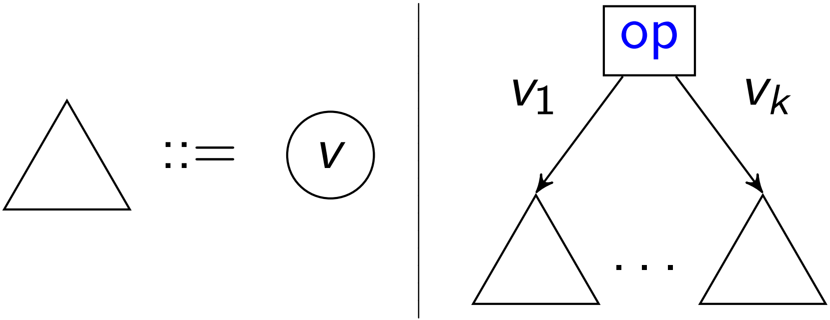


Example: nondeterminism (drunk coin toss)

$$\begin{aligned} \textsf{choose} &: \mathsf{Bool} \\ \textsf{fail} &: 0 \end{aligned}$$



What is an effectful computation?



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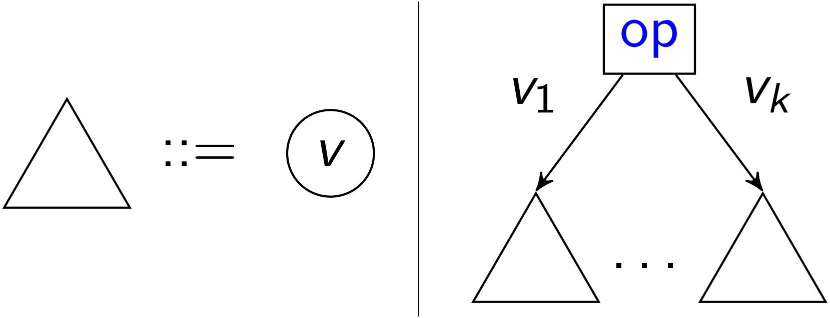

Equivalently (ignoring edge labels)

$$m ::= \textbf{return}\ v \mid \text{op}\ \langle m_1,\ \ldots,\ m_k \rangle$$

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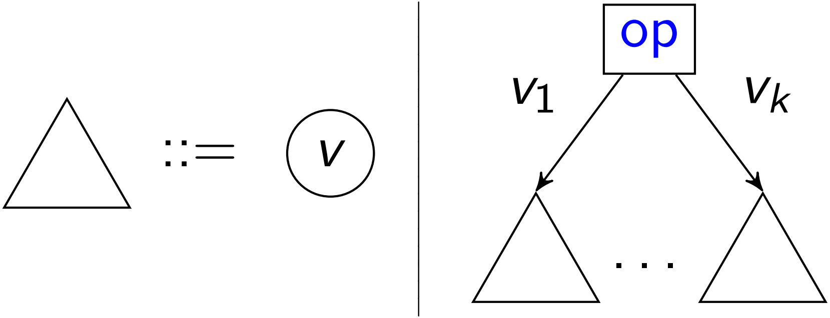

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$$m ::= \textbf{return}\, v \mid \textsf{op}\ (\lambda x.\textbf{case}\, x\, \{v_1 \mapsto m_1;\ \ldots;\ v_k \mapsto m_k\})$$

Examples

Boolean state

$$\mathsf{toggle} = \mathsf{get}\,\langle \mathsf{put}_{\mathsf{ff}}\,\langle \mathbf{return}\,\mathsf{tt}\rangle,\ \mathsf{put}_{\mathsf{tt}}\,\langle \mathbf{return}\,\mathsf{ff}\rangle\rangle$$

$$\mathbf{let}\ s = \mathsf{get}\,()\ \mathbf{in}\ \mathsf{put}\,(\mathsf{not}\ s);\ s$$

Examples

Boolean state

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Natural number state

$$\text{increment} = \text{get}\,\langle \text{put}_1\,\langle \textbf{return}\,()\rangle,\ \ldots,\ \text{put}_{i+1}\,\langle \textbf{return}\,()\rangle,\ \ldots\rangle$$

$$\text{put}\,(1 + \text{get}\,())$$

Examples

Boolean state

$$\mathsf{toggle} = \mathsf{get}\,\langle \mathsf{put}_{\mathsf{ff}}\,\langle \mathbf{return}\,\mathsf{tt}\rangle,\ \mathsf{put}_{\mathsf{tt}}\,\langle \mathbf{return}\,\mathsf{ff}\rangle\rangle$$

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Natural number state

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Nondeterminism

$$\mathsf{drunkToss} = \mathsf{choose}\,\langle \mathsf{choose}\,\langle \mathbf{return}\,\mathsf{Heads},\ \mathbf{return}\,\mathsf{Tails}\rangle,\ \mathsf{fail}\langle\rangle\rangle$$

$$\mathbf{if}\ \mathsf{choose}\,()\ \mathbf{then}\ (\mathbf{if}\ \mathsf{choose}\,()\ \mathbf{then}\ \mathsf{Heads}\ \mathbf{else}\ \mathsf{Tails})\ \mathbf{else}\ \mathbf{absurd}\ (\mathsf{fail}\,())$$

Command-response trees as free monads

- A computation of type comp A is a tree whose leaves have type A
- Return is **return**
- Bind perfoms substitution at the leaves

$$\begin{aligned} \textbf{return}\ v \gg\!= r &= r\ v \\ \mathsf{op}\ \langle m_1, \ldots, m_n \rangle \gg\!= r &= \mathsf{op}\ \langle m_1 \gg\!= r, \ldots, m_n \gg\!= r \rangle \end{aligned}$$

Algebraic effects

An algebraic effect is given by

1. a **signature** of operations
2. a collection of **equations**

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Example: boolean state

Signature

$$\begin{aligned} \mathsf{get} &: \mathsf{Bool} \\ \mathsf{put}_{\mathsf{tt}} &: 1 \\ \mathsf{put}_{\mathsf{ff}} &: 1 \end{aligned}$$

Algebraic effects

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1. a **signature** of operations
2. a collection of **equations**

Example: boolean state

Signature

$$\begin{aligned} \mathsf{get} &: \mathrm{Bool} \\ \mathsf{put}_{\mathsf{tt}} &: 1 \\ \mathsf{put}_{\mathsf{ff}} &: 1 \end{aligned}$$

Equations

$$\begin{aligned} \mathsf{put}_s \langle \mathsf{put}_{s'} \langle m \rangle \rangle &\simeq \mathsf{put}_{s'} \langle m \rangle && \text{(put-put)} \\ \mathsf{put}_s \langle \mathsf{get} \langle m_{\mathsf{tt}}, m_{\mathsf{ff}} \rangle \rangle &\simeq \mathsf{put}_s \langle m_s \rangle && \text{(put-get)} \\ \mathsf{get} \langle \mathsf{put}_{\mathsf{tt}} \langle m \rangle, \mathsf{put}_{\mathsf{ff}} \langle m \rangle \rangle &\simeq m && \text{(get-put)} \\ \mathsf{get} \langle \mathsf{get} \langle m, m' \rangle, \mathsf{get} \langle n', n \rangle \rangle &\simeq \mathsf{get} \langle m, n \rangle && \text{(get-get)} \end{aligned}$$

Aside: the (get-get) equation is redundant

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Interpreting algebraic effects

Example: boolean state

Standard interpretation ($[\![\mathrm{comp}\, A]\!] = \mathrm{Bool} \to [\![A]\!] \times \mathrm{Bool}$)

$$
\begin{aligned}
[\![\mathbf{return}\; v]\!] &= \lambda s.([\![v]\!], s) \\
[\![\mathsf{get}\,\langle m, n\rangle]\!] &= \lambda s.\mathbf{if}\; s\; \mathbf{then} [\![m]\!] s \;\mathbf{else}\; [\![n]\!] s \\
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\end{aligned}
$$

Logging interpretation ($[\![\mathrm{comp}\, A]\!] = \mathrm{Bool} \to [\![A]\!] \times \mathrm{List\,Bool}$)

$$
\begin{aligned}
[\![\mathbf{return}\ v]\!] &= \lambda s.([\![v]\!], [s]) \\
[\![\mathsf{get}\ \langle m, n\rangle]\!] &= \lambda s.\mathbf{if}\ s\ \mathbf{then}\,[\![m]\!]s\ \mathbf{else}\ [\![n]\!]s \\
[\![\mathsf{put}_{s'}\ \langle m\rangle]\!] &= \lambda s.\mathbf{let}\ (x, ss) \leftarrow [\![m]\!]s'\ \mathbf{in}\ (x, s :: ss)
\end{aligned}
$$

Example: boolean state, standard interpretation

$$
\begin{aligned}
[\![\mathrm{comp}\,A]\!] &= \mathrm{Bool} \to [\![A]\!] \times \mathrm{Bool} \\
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\end{aligned}
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Sound and complete with respect to the equations

$$m \simeq n \iff [\![m]\!] = [\![n]\!]$$

Bit toggling

$$[\![\mathrm{toggle}]\!] = \lambda s.\mathbf{if}\,s\,\mathbf{then}\,(\mathrm{tt}, \mathrm{ff})\,\mathbf{else}\,(\mathrm{ff}, \mathrm{tt})$$

Example: boolean state, discard interpretation

$$\begin{aligned}
[\![\mathsf{comp}\ A]\!] &= \mathsf{Bool} \to [\![A]\!] \\[1ex]
[\![\mathbf{return}\ v]\!] &= \lambda s.[\![v]\!] \\
[\![\mathsf{get}\ \langle m, n\rangle]\!] &= \lambda s.\mathbf{if}\ s\ \mathbf{then}\ [\![m]\!]s\ \mathbf{else}\ [\![n]\!]s \\
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\end{aligned}$$

Sound with respect to the equations

$$m \simeq n \implies [\![m]\!] = [\![n]\!]$$

Not complete because:

$$[\![\mathsf{put}_s\ \langle \mathbf{return}\ v\rangle]\!] = [\![\mathbf{return}\ v]\!]$$

Bit toggling

$$[\![\mathsf{toggle}]\!] = \lambda s.\mathbf{if}\ s\ \mathbf{then}\ \mathsf{tt}\ \mathbf{else}\ \mathsf{ff} = \lambda s.s$$

Example: boolean state, logging interpretation

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Complete with respect to the equations

$$m \simeq n \quad \Longleftarrow \quad [\![m]\!] = [\![n]\!]$$

Not sound because:

$$
\begin{aligned}
[\![\mathrm{put}_{s}\ \langle \mathrm{put}_{s'}\ \langle m\rangle\rangle]\!] &\neq [\![\mathrm{put}_{s'}\ \langle m\rangle]\!] \\
[\![\mathrm{get}\ \langle \mathrm{put}_{\mathrm{tt}}\ \langle m\rangle, \mathrm{put}_{\mathrm{ff}}\ \langle n\rangle\rangle]\!] &\neq [\![\mathrm{get}\ \langle m, n\rangle]\!]
\end{aligned}
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Bit toggling

$$[\![\mathrm{toggle}]\!] = \lambda s.\mathbf{if}\ s\ \mathbf{then}\,(\mathrm{tt}, [\mathrm{tt}, \mathrm{ff}])\ \mathbf{else}\,(\mathrm{ff}, [\mathrm{ff}, \mathrm{tt}])$$

Algebraic effects without equations

Different interpretations are useful in practice

So we will adopt **free** algebraic effects — no equations

Algebraic computations are command-response trees modulo equations

Abstract computations are plain command-response trees

Different interpretations give different meanings to the same abstract computation

Interpretations as effect handlers

Example: boolean state — standard interpretation

Meta level interpretation (enumerated continuations)

$$
\begin{aligned}
[\![\mathbf{return}\ v]\!] &= \lambda s.([\![v]\!], s) \\
[\![\mathsf{get}\ \langle m, n\rangle]\!] &= \lambda s.\mathbf{if}\ s\ \mathbf{then}\ [\![m]\!]s\ \mathbf{else}\ [\![n]\!]s \\
[\![\mathsf{put}_{s'}\ \langle m\rangle]\!] &= \lambda s.[\![m]\!]s'
\end{aligned}
$$

Interpretations as effect handlers

Example: boolean state — standard interpretation

Meta level interpretation (enumerated continuations)

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Meta level interpretation (continuations as functions)

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Object level effect handler

$$\begin{aligned} &\mathbf{return}\; v && \mapsto \lambda s.(v, s) \\ &\langle \mathsf{get}\; () \to r\rangle && \mapsto \lambda s.r\; s\; s \\ &\langle \mathsf{put}\; s' \to r\rangle && \mapsto \lambda s.r\; ()\; s' \end{aligned}$$

Interpretations as effect handlers

Example: nondeterminism

Meta level interpretation (enumerated continuations)

$$
\begin{aligned}
[\![\mathbf{return}\ v]\!] &= [[\![v]\!]] \\
[\![\mathsf{choose}\ \langle m, n\rangle]\!] &= [\![m]\!] +\!\!+ [\![n]\!] \\
[\![\mathsf{fail}\ \langle\rangle]\!] &= []
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$$

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Object level effect handler

$$\begin{aligned}
&\mathbf{return}\ v && \mapsto [v] \\
&\langle \mathsf{choose}\ () \to r\rangle && \mapsto r\ \mathsf{tt} \mathbin{+\!\!+} r\ \mathsf{ff} \\
&\langle \mathsf{fail}\ () \to r\rangle && \mapsto []
\end{aligned}$$

Parameterised operations (term parameters)

For convenience we write

$$\mathsf{op} : A \twoheadrightarrow B \qquad \text{instead of} \qquad \mathsf{op}_i : B, \quad i \in A$$

and to perform an operation:

$$\mathsf{op}\ i \qquad \text{instead of} \qquad \mathsf{op}_i$$

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For uniformity we parameterise all operations in effect signatures.

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For uniformity we parameterise all operations in effect signatures.

Examples

natural number state	$\{\mathsf{put} : \mathrm{Nat} \twoheadrightarrow 1, \quad \mathsf{get} : 1 \twoheadrightarrow \mathrm{Nat}\}$
boolean state	$\{\mathsf{put} : \mathrm{Bool} \twoheadrightarrow 1, \quad \mathsf{get} : 1 \twoheadrightarrow \mathrm{Bool}\}$
nondeterminism	$\{\mathsf{choose} : 1 \twoheadrightarrow \mathrm{Bool}, \quad \mathsf{fail} : 1 \twoheadrightarrow 0\}$

Parametric operations (type parameters)

It can be useful to parameterise an operation by one or more types.

Example: nondeterminism

$$\{\mathsf{choose} : 1 \twoheadrightarrow \mathsf{Bool}, \quad \mathsf{fail} : 1 \twoheadrightarrow 0\}$$

becomes

$$\{\mathsf{choose} : 1 \twoheadrightarrow \mathsf{Bool}, \quad \mathsf{fail} : a.1 \twoheadrightarrow a\}$$

and

$$\mathbf{if}\ \mathsf{choose}\,()\ \mathbf{then}\ (\mathbf{if}\ \mathsf{choose}\,()\ \mathbf{then}\ \mathsf{Heads}\ \mathbf{else}\ \mathsf{Tails})\ \mathbf{else}\ \mathbf{absurd}\ (\mathsf{fail}\,())$$

becomes

$$\mathbf{if}\ \mathsf{choose}\,()\ \mathbf{then}\ (\mathbf{if}\ \mathsf{choose}\,()\ \mathbf{then}\ \mathsf{Heads}\ \mathbf{else}\ \mathsf{Tails})\ \mathbf{else}\ \mathsf{fail}\,()$$

Example: choice and failure

Effect signature

$$\{\mathsf{choose} : 1 \twoheadrightarrow \mathsf{Bool}, \quad \mathsf{fail} : a.1 \twoheadrightarrow a\}$$

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$$\{\text{choose} : 1 \twoheadrightarrow \text{Bool}, \quad \text{fail} : a.1 \twoheadrightarrow a\}$$

Drunk coin tossing

$$\text{toss}\,() = \textbf{if } \text{choose}\,() \textbf{ then } \text{Heads} \textbf{ else } \text{Tails}$$

$$\begin{aligned}
\text{drunkToss}\,() = {} & \textbf{if } \text{choose}\,() \textbf{ then} \\
& \quad \textbf{if } \text{choose}\,() \textbf{ then } \text{Heads} \textbf{ else } \text{Tails} \\
& \textbf{else} \\
& \quad \text{fail}\,()
\end{aligned}$$

$$\begin{aligned}
\text{drunkTosses}\, n = {} & \textbf{if } n = 0 \textbf{ then } [] \\
& \textbf{else } \text{drunkToss}\,() :: \text{drunkTosses}\,(n-1)
\end{aligned}$$

Example: choice and failure

Handlers

$$
\begin{aligned}
&\mathrm{maybeFail} = \qquad \textcolor{red}{\text{— exception handler}}\\
&\quad \mathbf{return}\, x \;\mapsto\; \mathrm{Just}\, x\\
&\quad \langle \textcolor{blue}{\mathrm{fail}}\,()\rangle \;\mapsto\; \mathrm{Nothing}
\end{aligned}
$$

Example: choice and failure

Handlers

$$\text{maybeFail} = \quad \text{— exception handler}$$
$$\begin{aligned} \textbf{return}\, x &\mapsto \text{Just}\, x \\ \langle \text{fail}\,()\rangle &\mapsto \text{Nothing} \end{aligned}$$

$$\textbf{handle}\ \ 42\ \ \textbf{with}\ \text{maybeFail} \Longrightarrow \text{Just}\, 42$$
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trueChoice = — linear handler

$$\begin{aligned} &\textbf{return}\, x \mapsto x \\ &\langle \mathrm{choose}\,() \to r\rangle \mapsto r\, \mathrm{tt} \end{aligned}$$

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maybeFail $=$ — exception handler

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$$\textbf{handle}\ 42\ \textbf{with}\ \mathrm{trueChoice} \Longrightarrow 42$$

$$\textbf{handle}\ \mathrm{toss}\,()\ \textbf{with}\ \mathrm{trueChoice} \Longrightarrow \mathrm{Heads}$$

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Handlers

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$\mathrm{allChoices} = \quad$ — non-linear handler

$$\begin{aligned} &\mathbf{return}\, x \mapsto [x] \\ &\langle \mathrm{choose}\,() \to r\rangle \mapsto r\, \mathrm{tt} +\!\!+\, r\, \mathrm{ff} \end{aligned}$$

Example: choice and failure

Handlers

$$\begin{aligned}
&\mathrm{maybeFail} = \qquad \textcolor{red}{\text{— exception handler}}\\
&\quad \textbf{return}\, x \ \mapsto \mathrm{Just}\, x\\
&\quad \langle \textcolor{blue}{\mathrm{fail}}\,()\rangle \ \mapsto \mathrm{Nothing}
\end{aligned}$$

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&\textbf{handle}\ 42\ \textbf{with}\ \mathrm{maybeFail} \Longrightarrow \mathrm{Just}\, 42\\
&\textbf{handle}\ \textcolor{blue}{\mathrm{fail}}\,()\ \textbf{with}\ \mathrm{maybeFail} \Longrightarrow \mathrm{Nothing}
\end{aligned}$$

$$\begin{aligned}
&\mathrm{trueChoice} = \qquad \textcolor{red}{\text{— linear handler}}\\
&\quad \textbf{return}\, x \ \mapsto x\\
&\quad \langle \textcolor{blue}{\mathrm{choose}}\,() \to r\rangle \ \mapsto r\, \mathrm{tt}
\end{aligned}$$

$$\begin{aligned}
&\textbf{handle}\ 42\ \textbf{with}\ \mathrm{trueChoice} \Longrightarrow 42\\
&\textbf{handle}\ \mathrm{toss}\,()\ \textbf{with}\ \mathrm{trueChoice} \Longrightarrow \mathrm{Heads}
\end{aligned}$$

$$\begin{aligned}
&\mathrm{allChoices} = \qquad \textcolor{red}{\text{— non-linear handler}}\\
&\quad \textbf{return}\, x \ \mapsto [x]\\
&\quad \langle \textcolor{blue}{\mathrm{choose}}\,() \to r\rangle \ \mapsto r\, \mathrm{tt} \mathbin{+\!\!+} r\, \mathrm{ff}
\end{aligned}$$

$$\begin{aligned}
&\textbf{handle}\ 42\ \textbf{with}\ \mathrm{allChoices} \Longrightarrow [42]\\
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\end{aligned}$$

Example: choice and failure

Handlers

$\mathsf{maybeFail} = \quad$ — exception handler

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$$\textbf{handle}\ (\textbf{handle}\ \mathsf{drunkTosses}\, 2\ \textbf{with}\ \mathsf{maybeFail})\ \textbf{with}\ \mathsf{allChoices} \Longrightarrow$$

Example: choice and failure

Handlers

$$\begin{array}{ll}
\mathsf{maybeFail} = & \text{— exception handler} \\
\quad \mathbf{return}\, x \mapsto \mathsf{Just}\, x & \mathbf{handle}\ 42\ \mathbf{with}\ \mathsf{maybeFail} \Longrightarrow \mathsf{Just}\, 42 \\
\quad \langle \mathsf{fail}\, ()\rangle \mapsto \mathsf{Nothing} & \mathbf{handle}\ \mathsf{fail}\, ()\ \mathbf{with}\ \mathsf{maybeFail} \Longrightarrow \mathsf{Nothing}
\end{array}$$

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\mathsf{trueChoice} = & \text{— linear handler} \\
\quad \mathbf{return}\, x \mapsto x & \mathbf{handle}\ 42\ \mathbf{with}\ \mathsf{trueChoice} \Longrightarrow 42 \\
\quad \langle \mathsf{choose}\, () \to r\rangle \mapsto r\, \mathsf{tt} & \mathbf{handle}\ \mathsf{toss}\, ()\ \mathbf{with}\ \mathsf{trueChoice} \Longrightarrow \mathsf{Heads}
\end{array}$$

$$\begin{array}{ll}
\mathsf{allChoices} = & \text{— non-linear handler} \\
\quad \mathbf{return}\, x \mapsto [x] & \mathbf{handle}\ 42\ \mathbf{with}\ \mathsf{allChoices} \Longrightarrow [42] \\
\quad \langle \mathsf{choose}\, () \to r\rangle \mapsto r\, \mathsf{tt} \mathbin{+\!\!+} r\, \mathsf{ff} & \mathbf{handle}\ \mathsf{toss}\, ()\ \mathbf{with}\ \mathsf{allChoices} \Longrightarrow [\mathsf{Heads}, \mathsf{Tails}]
\end{array}$$

$$\begin{array}{l}
\mathbf{handle}\ (\mathbf{handle}\ \mathsf{drunkTosses}\, 2\ \mathbf{with}\ \mathsf{maybeFail})\ \mathbf{with}\ \mathsf{allChoices} \Longrightarrow \\
\quad [\mathsf{Just}\, [\mathsf{Heads}, \mathsf{Heads}],\ \mathsf{Just}\, [\mathsf{Heads}, \mathsf{Tails}],\ \mathsf{Nothing}, \\
\quad\ \ \mathsf{Just}\, [\mathsf{Tails}, \mathsf{Heads}],\ \mathsf{Just}\, [\mathsf{Tails}, \mathsf{Tails}],\ \mathsf{Nothing}, \\
\quad\ \ \mathsf{Nothing}]
\end{array}$$

Operational semantics

Reduction rules

$$\begin{aligned}
\textbf{let } x = V \textbf{ in } N &\rightsquigarrow N[V/x] \\
\textbf{handle } V \textbf{ with } H &\rightsquigarrow N[V/x] \\
\textbf{handle } \mathcal{E}[\mathsf{op}\ V] \textbf{ with } H &\rightsquigarrow N_{\mathsf{op}}[V/p,\ (\lambda x.\textbf{handle } \mathcal{E}[x] \textbf{ with } H)/r], \quad \mathsf{op} \mathbin{\#} \mathcal{E}
\end{aligned}$$

$$\begin{aligned}
\text{where } H = \textbf{return}\, x \quad &\mapsto N \\
\langle \mathsf{op}_1\, p \rightarrow r \rangle &\mapsto N_{\mathsf{op}_1} \\
&\cdots \\
\langle \mathsf{op}_k\, p \rightarrow r \rangle &\mapsto N_{\mathsf{op}_k}
\end{aligned}$$

Evaluation contexts

$$\mathcal{E} ::= [\,] \mid \textbf{let } x = \mathcal{E} \textbf{ in } N \mid \textbf{handle } \mathcal{E} \textbf{ with } H$$

Typing rules

Effects

$$E ::= \emptyset \mid \{\mathsf{op} : A \twoheadrightarrow B\} \uplus E$$

Computations

$$C, D ::= A \,!\, E$$

Operations

$$\frac{\Gamma \vdash V : A}{\Gamma \vdash \mathsf{op}\ V : B \,!\, (\{\mathsf{op} : A \twoheadrightarrow B\} \uplus E)}$$

Handlers

$$\frac{\Gamma \vdash M : C \qquad \Gamma \vdash H : C \Rightarrow D}{\Gamma \vdash \textbf{handle}\ M\ \textbf{with}\ H : D}$$

$$\frac{\Gamma, x : A \vdash N : D \qquad [\mathsf{op}_i : A_i \twoheadrightarrow B_i \in E]_i \qquad [\Gamma, p : A_i, r : B_i \to D \vdash N_i : D]_i}{\Gamma \vdash \begin{array}{l}\textbf{return}\ x \mapsto N \\ (\langle \mathsf{op}_i\ p \to r\rangle \mapsto N_i)_i\end{array} : A \,!\, E \Rightarrow D}$$

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Exercise: Adapt the typing rules to accommodate parametric operations

What is an effect handler?

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- A **modular** interpreter for effectful computations

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- A generalisation of an exception handler
  - based on exceptional syntax [Benton and Kennedy, 2001]

$$\begin{array}{cl} \textbf{let}\ x = (\textbf{try}\ M\ \textbf{with}\ H)\ \textbf{in}\ N & \text{conventional exception handlers} \\ \Downarrow & \\ \textbf{try}\ M\ \textbf{as}\ x\ \textbf{in}\ N\ \textbf{otherwise}\ H & \text{exceptional syntax} \\ \Downarrow & \\ \textbf{handle}\ M\ \textbf{with}\ \{\textbf{return}\ x \mapsto N; H\} & \text{effect handlers} \end{array}$$

  success continuations aid composition, optimisation, and reasoning
  - resumable

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- A morphism between (free) algebras
- A fold (catamorphism) over a command-response tree
- A structured delimited control operator
- A composable user-defined operating system

Example: state

Effect signature

$$\{\mathsf{put} : \mathsf{Nat} \twoheadrightarrow 1, \quad \mathsf{get} : 1 \twoheadrightarrow \mathsf{Nat}\}$$

Example: state

Effect signature

$$\{\mathsf{put} : \mathsf{Nat} \twoheadrightarrow 1, \quad \mathsf{get} : 1 \twoheadrightarrow \mathsf{Nat}\}$$

Standard state handler

$$\begin{array}{ll}
\mathsf{state} = & \text{— closure handler} \\
\quad \mathbf{return}\, v & \mapsto \lambda s.(v, s) \\
\quad \langle \mathsf{get}\, () \to r \rangle & \mapsto \lambda s.r\, s\, s \\
\quad \langle \mathsf{put}\, s' \to r \rangle & \mapsto \lambda s.r\, ()\, s'
\end{array}$$

Example: state

Effect signature

$$\{\mathsf{put} : \mathsf{Nat} \twoheadrightarrow 1, \quad \mathsf{get} : 1 \twoheadrightarrow \mathsf{Nat}\}$$

Standard state handler

$$\begin{array}{lll}
\mathsf{state}'\,(s) = & & \text{— parameterised handler} \\
\quad \mathbf{return}\; v & \mapsto & (v, s) \\
\quad \langle \mathsf{get}\,() \to r\rangle & \mapsto & r\;s\;s \\
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\end{array}$$

$$\mathbf{handle}\; \mathsf{put}\,(1 + \mathsf{get}\,())\; \mathbf{with}\; \mathsf{state}'\,(42) \Longrightarrow$$

Example: state

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\end{array}$$

$$\begin{array}{l}
\mathbf{handle}\ \mathsf{put}\,(1 + \mathsf{get}\,())\ \mathbf{with}\ \mathsf{state}'\,(42) \Longrightarrow \\
\quad ((), 43)
\end{array}$$

Operational semantics (parameterised handlers)

Reduction rules

$$\begin{array}{rcl}
\textbf{let}\ x = V\ \textbf{in}\ N & \rightsquigarrow & N[V/x] \\
\textbf{handle}\ V\ \textbf{with}\ H\ W & \rightsquigarrow & N[V/x, W/h] \\
\textbf{handle}\ \mathcal{E}[\mathsf{op}\ V]\ \textbf{with}\ H\ W & \rightsquigarrow & N_{\mathsf{op}}[V/p,\ W/h,\ (\lambda h\, x.\textbf{handle}\ \mathcal{E}[x]\ \textbf{with}\ H\ h)/r], \quad \mathsf{op} \# \mathcal{E}
\end{array}$$

$$\begin{array}{rrcl}
\text{where } H\ h = & \textbf{return}\, x & \mapsto & N \\
& \langle \mathsf{op}_1\ p \to r \rangle & \mapsto & N_{\mathsf{op}_1} \\
& & \dots & \\
& \langle \mathsf{op}_k\ p \to r \rangle & \mapsto & N_{\mathsf{op}_k}
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Evaluation contexts

$$\mathcal{E} ::= [\,] \mid \textbf{let}\ x = \mathcal{E}\ \textbf{in}\ N \mid \textbf{handle}\ \mathcal{E}\ \textbf{with}\ H\ W$$

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\textbf{let } x = V \textbf{ in } N &\rightsquigarrow N[V/x] \\
\textbf{handle } V \textbf{ with } H\ W &\rightsquigarrow N[V/x, W/h] \\
\textbf{handle } \mathcal{E}[\mathsf{op}\ V] \textbf{ with } H\ W &\rightsquigarrow N_{\mathsf{op}}[V/p,\ W/h,\ (\lambda h\, x.\textbf{handle } \mathcal{E}[x] \textbf{ with } H\ h)/r], \quad \mathsf{op} \# \mathcal{E}
\end{aligned}
$$

$$
\begin{aligned}
\text{where } H\ h = \textbf{return}\, x \quad &\mapsto N \\
\langle \mathsf{op}_1\ p \to r \rangle &\mapsto N_{\mathsf{op}_1} \\
&\dots \\
\langle \mathsf{op}_k\ p \to r \rangle &\mapsto N_{\mathsf{op}_k}
\end{aligned}
$$

Evaluation contexts

$$\mathcal{E} ::= [\,] \mid \textbf{let } x = \mathcal{E} \textbf{ in } N \mid \textbf{handle } \mathcal{E} \textbf{ with } H\ W$$

Exercise: express parameterised handlers as deep handlers

Typing rules (parameterised handlers)

Effects

$$E ::= \emptyset \mid \{\mathsf{op} : A \twoheadrightarrow B\} \uplus E$$

Computations

$$C, D ::= A \,!\, E$$

Operations

$$\frac{\Gamma \vdash V : A}{\Gamma \vdash \mathsf{op}\; V : B \,!\, (\{\mathsf{op} : A \twoheadrightarrow B\} \uplus E)}$$

Handlers

$$\frac{\Gamma \vdash M : C \qquad \Gamma \vdash V : P \qquad \Gamma \vdash H : P \to C \Rightarrow D}{\Gamma \vdash \mathbf{handle}\; M \;\mathbf{with}\; H\; V : D}$$

$$\frac{\begin{array}{c}\Gamma, h : P, x : A \vdash N : D \\ [\mathsf{op}_i : A_i \twoheadrightarrow B_i \in E]_i \qquad [\Gamma, h : P, p : A_i, r : P \to B_i \to D \vdash N_i : D]_i\end{array}}{\Gamma \vdash \begin{array}{l}\lambda h.\mathbf{return}\; x \mapsto N \\ \quad (\langle \mathsf{op}_i\; p \to r\rangle \mapsto N_i)_i\end{array} : P \to A \,!\, E \Rightarrow D}$$

Example: cooperative lightweight threads

Effect signature

$$\begin{aligned}\mathsf{Co} = \{&\mathsf{yield} : 1 \twoheadrightarrow 1,\\ &\mathsf{fork} : \mathsf{Thread} \twoheadrightarrow 1\}\end{aligned}$$

$$\mathsf{Thread} = 1 \to 1 \,!\, \mathsf{Co}$$

Example: cooperative lightweight threads

Effect signature

$$\begin{aligned}\mathsf{Co} = \{\mathsf{yield} &: 1 \twoheadrightarrow 1,\\ \mathsf{fork} &: \mathsf{Thread} \twoheadrightarrow 1\}\end{aligned}$$

$$\mathsf{Thread} = 1 \to 1 \,!\, \mathsf{Co}$$

A single cooperative program

$$\begin{aligned}&\mathsf{main} : \mathsf{Thread}\\ &\mathsf{main}\,() = \mathsf{print}\,(\text{“M1 ”}); \mathsf{fork}\,(\textbf{fun}\,() \to \mathsf{print}\,(\text{“A1 ”}); \mathsf{yield}\,(); \mathsf{print}\,(\text{“A2 ”}));\\ &\qquad\qquad \mathsf{print}\,(\text{“M2 ”}); \mathsf{fork}\,(\textbf{fun}\,() \to \mathsf{print}\,(\text{“B1 ”}); \mathsf{yield}\,(); \mathsf{print}\,(\text{“B2 ”}));\\ &\qquad\qquad \mathsf{print}\,(\text{“M3 ”})\end{aligned}$$

Example: cooperative lightweight threads

Handler

$$
\begin{array}{l}
\mathsf{cooperate} : \mathsf{List}\,(\mathsf{Thread}) \to 1 \\
\mathsf{cooperate}\,[] = () \\
\mathsf{cooperate}\,(t :: ts) = \\
\quad \textbf{handle}\; t()\; \textbf{with} \quad \textcolor{red}{\text{— shallow handler}} \\
\qquad \begin{array}{ll}
\textbf{return}\,() & \mapsto \mathsf{cooperate}\,(ts) \\
\langle \textcolor{blue}{\mathsf{yield}}\,() \to t \rangle & \mapsto \mathsf{cooperate}\,(ts +\!\!+ [t]) \\
\langle \textcolor{blue}{\mathsf{fork}}\; t \to r \rangle & \mapsto \mathsf{cooperate}\,(ts +\!\!+ [t, r])
\end{array}
\end{array}
$$

Example: cooperative lightweight threads

Handler

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\mathrm{cooperate} : \mathrm{List}\,(\mathrm{Thread}) \to 1 \\
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\langle \mathrm{fork}\ t \to r \rangle & \mapsto \mathrm{cooperate}\,(ts \mathbin{+\!\!+} [t, r])
\end{array}
\end{array}
$$

$$\mathrm{cooperate}\,[\mathrm{main}] \implies ()$$

M1 A1 M2 A2 B1 M3 B2

Example: cooperative lightweight threads

Handler

$$
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\mathsf{cooperate} : \mathsf{List}\,(\mathsf{Thread}) \to 1 \\
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\langle \mathsf{fork}\; t \to r \rangle & \mapsto \mathsf{cooperate}\,(ts +\!\!+ [t, r])
\end{array}
\end{array}
$$

Example: cooperative lightweight threads

Handler

$$
\begin{array}{l}
\mathsf{cooperate} : \mathsf{List}\,(\mathsf{Thread}) \to 1 \\
\mathsf{cooperate}\,[\,] = (\,) \\
\mathsf{cooperate}\,(t :: ts) = \\
\quad \textbf{handle}\; t(\,)\; \textbf{with} \quad \textcolor{red}{\text{— shallow handler}} \\
\qquad \begin{array}{lll}
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\langle \textcolor{blue}{\mathsf{fork}}\; t \to r \rangle & \mapsto & \mathsf{cooperate}\,(ts \mathbin{+\!\!+} [r, t])
\end{array}
\end{array}
$$

Example: cooperative lightweight threads

Handler

$$
\begin{array}{l}
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\end{array}
\end{array}
$$

$$\text{cooperate}\,[\text{main}] \implies ()$$

M1 M2 M3 A1 B1 A2 B2

Example: cooperative lightweight threads

Handler

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\end{array}
\end{array}
$$

$$
\begin{array}{l}
\mathsf{cooperate}\,[\mathsf{main}] \implies (\,) \\
\text{M1 M2 M3 A1 B1 A2 B2}
\end{array}
$$

Exercise: reimplement cooperate with parameterised handlers

Operational semantics (shallow handlers)

Reduction rules

$$\begin{aligned}
\textbf{let}\ x = V\ \textbf{in}\ N &\leadsto N[V/x] \\
\textbf{handle}\ V\ \textbf{with}\ H &\leadsto N[V/x] \\
\textbf{handle}\ \mathcal{E}[\mathsf{op}\ V]\ \textbf{with}\ H &\leadsto N_{\mathsf{op}}[V/p, (\lambda x.\mathcal{E}[x])/r], \qquad \mathsf{op} \# \mathcal{E}
\end{aligned}$$

$$\begin{aligned}
\text{where } H = \textbf{return}\, x \quad &\mapsto N \\
\langle \mathsf{op}_1\ p \to r \rangle &\mapsto N_{\mathsf{op}_1} \\
&\cdots \\
\langle \mathsf{op}_k\ p \to r \rangle &\mapsto N_{\mathsf{op}_k}
\end{aligned}$$

Evaluation contexts

$$\mathcal{E} ::= [\,] \mid \textbf{let}\ x = \mathcal{E}\ \textbf{in}\ N \mid \textbf{handle}\ \mathcal{E}\ \textbf{with}\ H$$

Operational semantics (shallow handlers)

Reduction rules

$$\begin{aligned}
\textbf{let } x = V \textbf{ in } N &\rightsquigarrow N[V/x] \\
\textbf{handle } V \textbf{ with } H &\rightsquigarrow N[V/x] \\
\textbf{handle } \mathcal{E}[\mathsf{op}\ V] \textbf{ with } H &\rightsquigarrow N_{\mathsf{op}}[V/p, (\lambda x.\mathcal{E}[x])/r], \quad \mathsf{op} \# \mathcal{E}
\end{aligned}$$

$$\begin{aligned}
\text{where } H = \textbf{return}\, x &\mapsto N \\
\langle \mathsf{op}_1\ p \to r \rangle &\mapsto N_{\mathsf{op}_1} \\
&\dots \\
\langle \mathsf{op}_k\ p \to r \rangle &\mapsto N_{\mathsf{op}_k}
\end{aligned}$$

Evaluation contexts

$$\mathcal{E} ::= [\ ] \mid \textbf{let } x = \mathcal{E} \textbf{ in } N \mid \textbf{handle } \mathcal{E} \textbf{ with } H$$

Exercise: express shallow handlers as deep handlers

Typing rules (shallow handlers)

Effects

$$E ::= \emptyset \mid \{\mathsf{op} : A \twoheadrightarrow B\} \uplus E$$

Computations

$$C, D ::= A \,!\, E$$

Operations

$$\frac{\Gamma \vdash V : A}{\Gamma \vdash \mathsf{op}\ V : B \,!\, (\{\mathsf{op} : A \twoheadrightarrow B\} \uplus E)}$$

Handlers

$$\frac{\Gamma \vdash M : C \qquad \Gamma \vdash H : C \Rightarrow D}{\Gamma \vdash \textbf{handle}\ M\ \textbf{with}\ H : D}$$

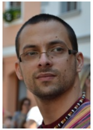
$$\frac{\Gamma, x : A \vdash N : D \qquad [\mathsf{op}_i : A_i \twoheadrightarrow B_i \in E]_i \qquad [\Gamma, p : A_i, r : B_i \to A \,!\, E \vdash N_i : D]_i}{\Gamma \vdash \begin{array}{l}\textbf{return}\ x \mapsto N \\ (\langle \mathsf{op}_i\ p \to r\rangle \mapsto N_i)_i\end{array} : A \,!\, E \Rightarrow D}$$

Effect handler oriented programming languages

Eff	https://www.eff-lang.org/
Effekt	https://effekt-lang.org/
Frank	https://github.com/frank-lang/frank
Helium	https://bitbucket.org/pl-uwr/helium
MET	https://github.com/4y8/modal-effect-types
Links	https://www.links-lang.org/
Koka	https://github.com/koka-lang/koka
OCaml 5	https://github.com/ocamllabs/ocaml-multicore/wiki
Unison	https://www.unison-lang.org/

Resources

Jeremy Yallop's effects bibliography
https://github.com/yallop/effects-bibliography



Matija Pretnar's tutorial
"An introduction to algebraic effects and handlers", MFPS 2015

Daniel Hillerström's PhD thesis
"Foundations for programming and implementing effect handlers", 2022

EHOP web page
https://effect-handlers.org/