Tracking Linear Continuations for Effect Handlers

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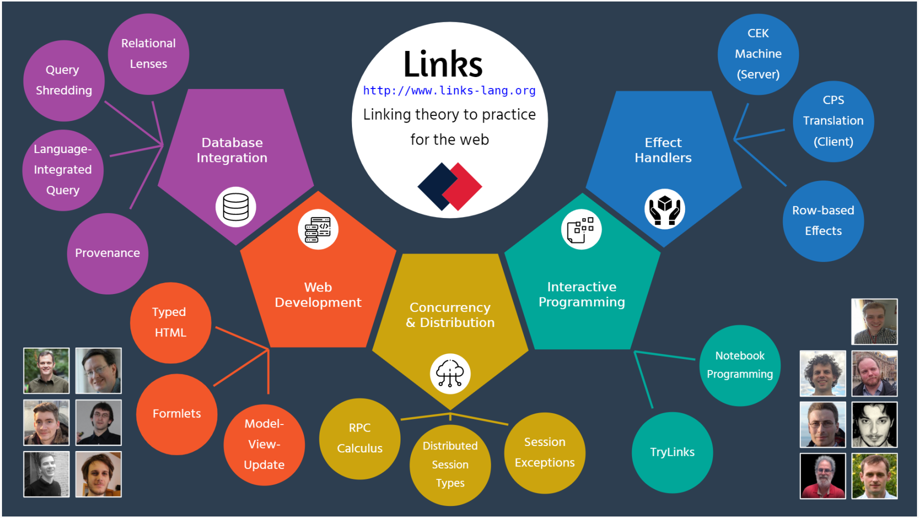
(Joint work with Daniel Hillerström, J. Garrett Morris, and Sam Lindley)

Links



Picture by Simon Fowler

Linear Types in Links

Links uses linear types for session types:

- `!A.S` : send a value of type `A`, then continue as `S`
- `?A.S` : receive a value of type `A`, then continue as `S`
- `End` : no communication

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Primitive operations on session-typed channels:

```
send    : ∀ a (b::Session) . (a, !a.b) → b
receive : ∀ a (b::Session) . (?a.b) → (a, b)
fork    : ∀   (a::Session) . (a → ()) → ~a
close   : End → ()
```

Linear Types in Links

A sender sends an integer.

```
sig sender       : (!Int.End) ⟶ ()
fun sender(ch)   { var ch' = send(42, ch); close(ch') }
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A receiver receives the integer and prints it.

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sig receiver    : (?Int.End) → ()
fun receiver(ch) { var (i, ch') = receive(ch); close(ch'); printInt(i) }
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fun receiver(ch) { var (i, ch') = receive(ch); close(ch'); printInt(i) }
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Fork the receiver and pass the dual channel to the sender.

```
links> { var ch = fork(receiver); sender(ch) };
42
```

Well-Typed Programs in Links Cannot Go Wrong

Linear types prevent us from using the same channel twice.

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links> { var ch = fork(receiver); sender(ch); sender(ch); };
Type error: Variable ch has linear type `!Int.End'
but is used 2 times.
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Or in a linear function.

```
links> { var ch = fork(receiver);
         var f  = linfun(){ sender(ch) }; f(); f() };
<stdin>:1: Type error: Variable f has linear type `() -@ ()'
but is used 2 times.
```

Effect Handlers in Links

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links> handle (choose())
         { case <Choose ⇒ r> → r(true) }
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Well-Typed Programs in Links CAN Go Wrong ! [1] [2]

We can use the same channel twice by invoking the continuation twice.

```
links> handle
       ({ var ch = fork(receiver); var _ = do Choose; sender(ch) })
       { case <Choose => r> -> r(true); r(false) }
```

[1] https://github.com/links-lang/links/issues/544

[2] Emrich and Hillerström, "Broken Links (Presentation)", 2020.

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The problem is that the continuation has an unlimited type `r : Bool → ()`, which does not reflect the usage of the linear channel `ch`.

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Our Solution

The previous type-and-effect system does not track linear continuations.

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links> fun(){var ch = fork(receiver); var _ = do Choose; sender(ch)};
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Use the operation signatures to track linear continuations.

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links> fun(){var ch = fork(receiver); var _ = lindo Choose; sender(ch)};
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        ({ var ch = fork(receiver); var _ = lindo Choose; sender(ch) })
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Now we know the continuation has a linear type `r : Bool -@ ()`.

Core Ideas of Type Checking

Consider two sequenced computations

$$\underbrace{M}_{A\,!\,\{R_1\}}\;;\;\underbrace{N}_{B\,!\,\{R_2\}} : B\,!\,\{R\}$$

We make sure R_1 is linear if N uses any linear resources.

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$$\underbrace{M}_{A\,!\,\{R_1\}} ; \underbrace{N}_{B\,!\,\{R_2\}} : B\,!\,\{R\}$$

We make sure R_1 is linear if N uses any linear resources.

For instance,

$$\underbrace{\mathbf{do}\ \mathit{Choose}}_{\mathit{Bool}\,!\,\{\mathit{Choose}:\text{linear}\}} ; \underbrace{\mathit{sender}(\mathit{ch})}_{()\,!\,\{R_2\}} : B\,!\,\{R\}$$

Core Ideas of Type Checking

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What's the relationship between R_1, R_2 and R?

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What's the relationship between R_1, R_2 and R?

- Row polymorphism: $R_1 = R_2 = R$
- Row subtyping: $R_1 \leqslant R, R_2 \leqslant R$
- Algebraic row subtyping: $R = R_1 \sqcup R_2$

The conventional effect system based on row polymorphism is too coarse for tracking linear continuations, because when N uses linear resources we only need to guarantee that operations in R_1 are linear.

Core Ideas of Type Inference

We make sure R_1 is linear if N uses any linear resources.

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However, we do not always know whether N uses any linear resources during the type inference.

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However, we do not always know whether N uses any linear resources during the type inference.

Add linearity annotations to sequencing (as well as operation invocations and handler clauses).

We force R_1 to be linear when

$$\underbrace{M}_{A\,!\,\{R_1\}} ;^{\circ} \underbrace{N}_{B\,!\,\{R_2\}} : B\,!\,\{R\}$$

We force free variables in N to be unlimited when

$$\underbrace{M}_{A\,!\,\{R_1\}} ;^{\bullet} \underbrace{N}_{B\,!\,\{R_2\}} : B\,!\,\{R\}$$

Core Ideas of Type Inference

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Qualified types / type inference with constraints.

$$\underbrace{M}_{A\,!\,\{R_1\}}\,;\,\underbrace{N}_{B\,!\,\{R_2\}} : B\,!\,\{R\} \mid (N \text{ contains free linear vars} \Rightarrow R_1 \text{ is linear})$$

We can also add the subtyping constraints.

$$B\,!\,\{R\} \mid (N \text{ contains free linear vars} \Rightarrow R_1 \text{ is linear}) \wedge (R_1 \leqslant R) \wedge (R_2 \leqslant R)$$

Conclusion

Our main contributions:

- F°_{eff}: a fine-grained call-by-value variant of *system F* with correct interaction between linear types and effect handlers.

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- An implementation of F°_{eff} in Links with *ML-style type inference* which requires a fair amount of linearity annotations.

Conclusion

Our main contributions:

- $\mathrm{F}^{\circ}_{\mathrm{eff}}$: a fine-grained call-by-value variant of *system F* with correct interaction between linear types and effect handlers.
- An implementation of $\mathrm{F}^{\circ}_{\mathrm{eff}}$ in Links with *ML-style type inference* which requires a fair amount of linearity annotations.
- $\mathrm{Q}^{\circ\leqslant}_{\mathrm{eff}}$: a *ML-style calculus* with linear types and effect subtyping based on *qualified types*. It requires no syntactic overheads and has better accuracy on tracking linear continuations.

Thank you!