Asymptotic Improvement through Delimited Control

Fast Generic Search with Effect Handlers

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(Joint work with Sam Lindley and John Longley)

The Fundamental Efficiency of Effect Handlers

We consider whether a language with effect handlers admit essential expressiveness differences over a "pure" language.

The question

Let $\mathcal{L}_{\mathit{eff}}$ a language with effect handlers, and $\mathcal{L} \subset \mathcal{L}_{\mathit{eff}}$ the fragment modulo effect handlers. Does $\mathcal{L}_{\mathit{eff}}$ admit asymptotically more efficient programs than $\mathcal{L}$?

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Spoiler alert: the answer is **YES**. Specifically $\mathcal{O}(2^n)$ vs $\Omega(n2^n)$.

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Take $\mathcal{L}$ to be cbv PCF and endow it with effect handlers to obtain $\mathcal{L}_{\mathit{eff}}$.

The Generic Search Problem

Problem: Given a boolean-valued predicate P on a space $\mathbb{B}^n$ of boolean vectors of length n (for some fixed $n \in \mathbb{N}$), return the number of such vectors p for which $P(p) = \mathrm{true}$. Thus for each n, we ask for an implementation of

$$\mathrm{count}_n : ((\mathrm{Nat} \to \mathrm{Bool}) \to \mathrm{Bool}) \to \mathrm{Nat}$$

There is but one rule:

No change of types is allowed! (Longley and Normann 2015)

This rules out tricks such as

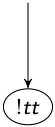
- CPS conversion
- Implementing an interpreter for $\mathcal{L}_{eff}$ in $\mathcal{L}$

Example predicates and their models I

A boring constant predicate

$$
\begin{aligned}
tt_0 &: (\mathsf{Nat} \to \mathsf{Bool}) \to \mathsf{Bool} \\
tt_0 &\doteq \lambda p.\mathsf{true}
\end{aligned}
$$

Admits a with no queries model

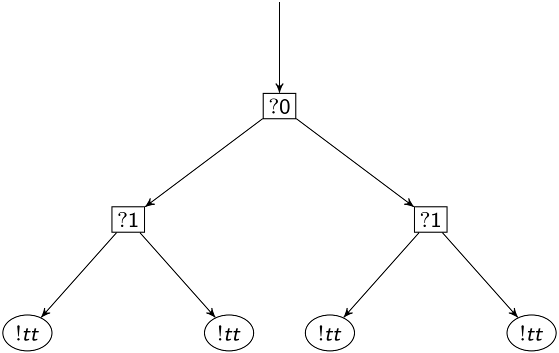


Example predicates and their models II

A slightly more interesting constant predicate

$$
\begin{aligned}
tt_2 &: (\mathsf{Nat} \to \mathsf{Bool}) \to \mathsf{Bool} \\
tt_2 &\doteq \lambda p.p\,0; p\,1; \mathsf{true}
\end{aligned}
$$

Admits a finite model with no repeated queries

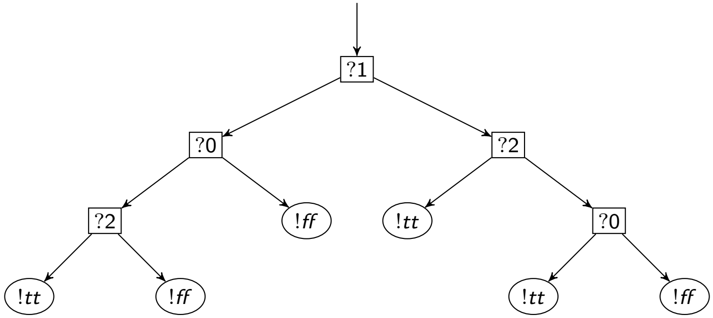


Example predicates and their models III

A non-constant predicate

$$
\begin{aligned}
tf_3 &\;:\; (\mathsf{Nat} \to \mathsf{Bool}) \to \mathsf{Bool} \\
tf_3 &\doteq \lambda p.\textbf{if } p\ 1 \\
&\qquad\quad \textbf{then if } p\ 0 \textbf{ then } p\ 2 \textbf{ else } \mathsf{false} \\
&\qquad\quad \textbf{else if } p\ 2 \textbf{ then } \mathsf{true} \textbf{ else } p\ 0
\end{aligned}
$$

Admits a finite model with no repeated queries

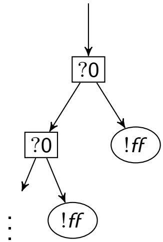


Example predicates and their models IV

Possibly divergent predicate

$$
\begin{aligned}
\mathit{div}_0 &: (\mathsf{Nat} \to \mathsf{Bool}) \to \mathsf{Bool} \\
\mathit{div}_0 &\doteq \mathbf{rec}\ \mathit{div}_0\ p.\mathbf{if}\ p\,0\ \mathbf{then}\ \mathit{div}_0\ p\ \mathbf{else}\ \mathsf{false}
\end{aligned}
$$

Admits an infinite model with repeated queries



Restriction to n-standard predicates

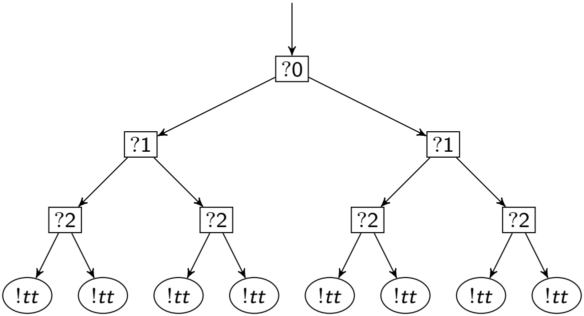
We restrict our analysis to predicates whose models are "n-standard"; informally

- A perfect binary tree of height $n > 0$, whose interior nodes are queries and leaves are answers.
- Contains every query $?j$ for $j \in \{0, \ldots, n-1\}$.
- No repeated queries along any path in the model.

For example

$$tt_3 \doteq \lambda p.p\ 0; p\ 1; p\ 2; \mathsf{true}$$

is 3-standard because its model is 3-standard



A pure generic search procedure

A possible implementation of generic search in $\mathcal{L}$

$$
\begin{array}{l}
\mathit{count}_n : ((\mathsf{Nat} \to \mathsf{Bool}) \to \mathsf{Bool}) \to \mathsf{Nat} \\
\mathit{count}_n \doteq \lambda \mathit{pred}.\mathit{count}'\, n\, (\lambda i.\bot) \\
\qquad\qquad \textbf{where} \\
\qquad\qquad\qquad \begin{array}{lcl}
\mathit{count}'\, 0 \quad\quad\; p & \doteq & \textbf{if } \mathit{pred}\, p \textbf{ then } 1 \textbf{ else } 0 \\
\mathit{count}'\, (1+n)\; p & \doteq & \quad \mathit{count}'\, n\, (\lambda i.\textbf{if } i = n \textbf{ then } \mathsf{true} \textbf{ else } p\, i) \\
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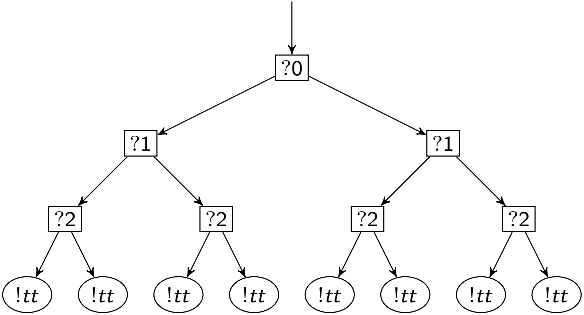

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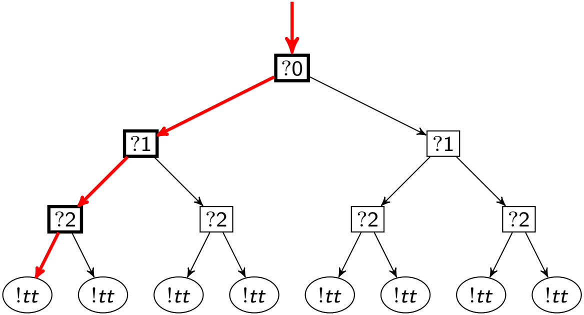
Example $\mathit{count}_3\ \mathit{tt}_3$: reaches the first leaf



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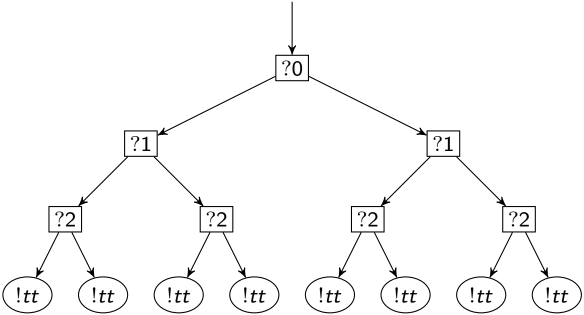

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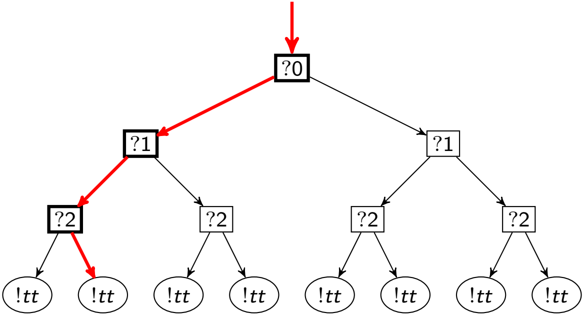
Example $count_3\, tt_3$: reaches the second leaf



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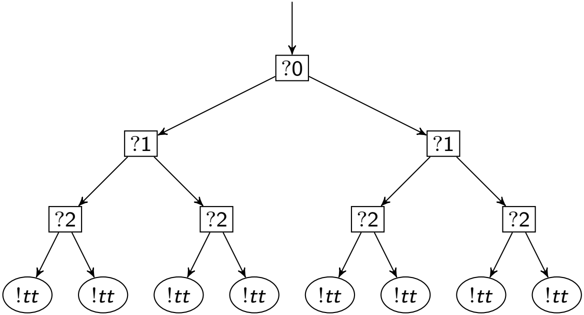

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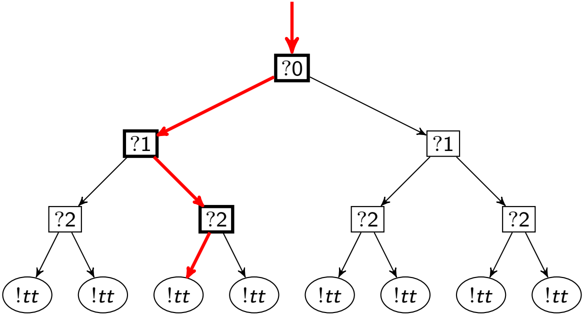
Example $\mathit{count}_3\ \mathit{tt}_3$: reaches the third leaf, etc. . .



The effectful generic search procedure

For the efficient implementation of generic search in $\mathcal{L}_{\mathit{eff}}$, we require one operation; fix $\Sigma \doteq \{\mathsf{Branch} : \langle\rangle \to \mathsf{Bool}\}$

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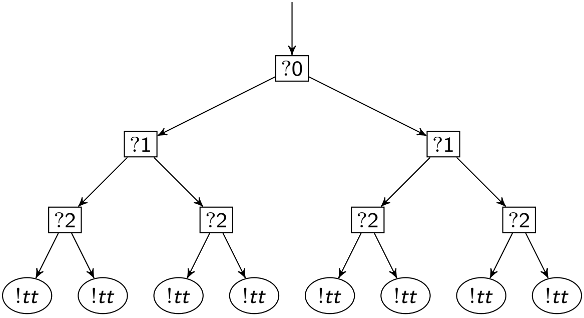

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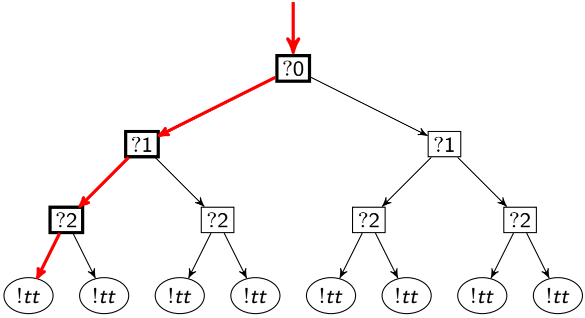

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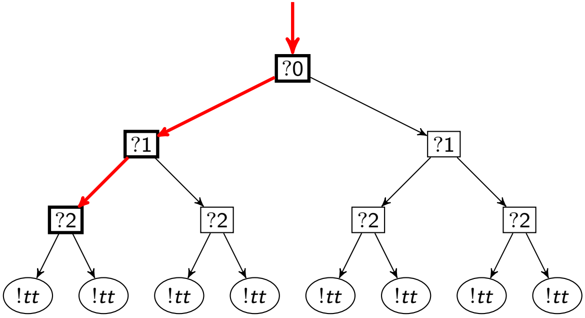
Example $\mathsf{count}\ tt_3$: computation backtracks



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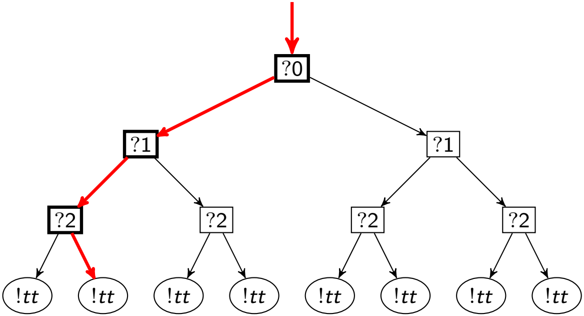

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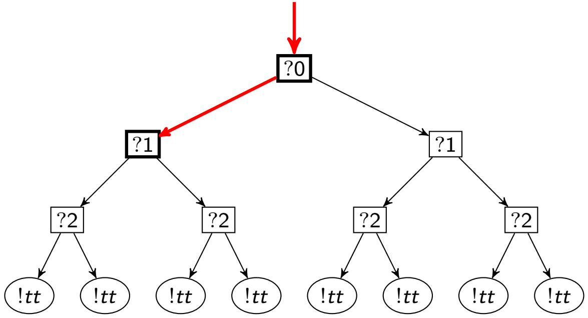

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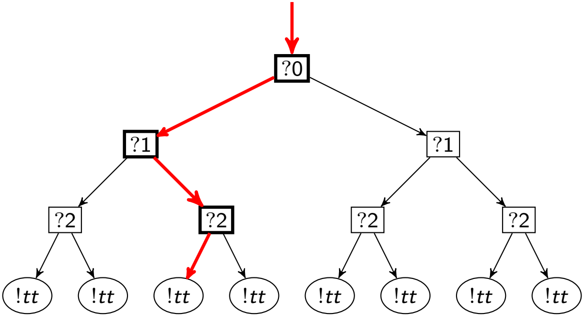


Main theorem

Theorem

1. *For every n-standard predicate pred, the generic counting procedure has at most time complexity*

$$\mathrm{DTime}(\mathrm{count}\ \mathit{pred}) = \sum_{bs \in \mathbb{B}^*, |bs| \leq n} \mathrm{steps}(t)(bs) + \mathcal{O}(2^n)$$

2. *Every generic counting function count $\in \mathcal{L}$ has for every n-standard predicate pred at least time complexity*

$$\mathrm{DTime}(\mathit{count}\ \mathit{pred}) = \sum_{bs \in \mathbb{B}^*, |bs| \leq n} 2^{n-|bs|}\mathrm{steps}(t)(bs) + \mathcal{O}(n2^n)$$

Here t denotes the model of pred, and $\mathrm{steps}(t)(bs)$ *computes the number of reduction steps used to arrive at the query or answer node determined by bs.*

Proving the positive result

Define suitable machine configuration computing functions

$$\mathrm{arrive}, \mathrm{depart} : \mathbb{B}^* \times \mathrm{Model} \rightharpoonup \mathrm{Conf}$$

Lemma

Suppose t is a model of a n-standard predicate, then for every boolean list $bs \in \mathbb{B}^*$

$$\mathrm{arrive}(bs, t) \longrightarrow^{\sum_{|bs| \leq n} \mathsf{steps}(t)(bs) + 2^{n-|bs|}} \mathrm{depart}(bs, t)$$

Proof.

Proof by downward induction on the list of booleans bs. □

Proving the negative result

Suppose that we have an arbitrary implementation of generic search $\textit{count} \in \mathcal{L}$. Pick any n-standard predicate *pred* and look at the computation arising from *count pred*. Now we need to show that

Lemma (Every leaf is visited (A))

The computation (count pred) visits every leaf in the model of pred.

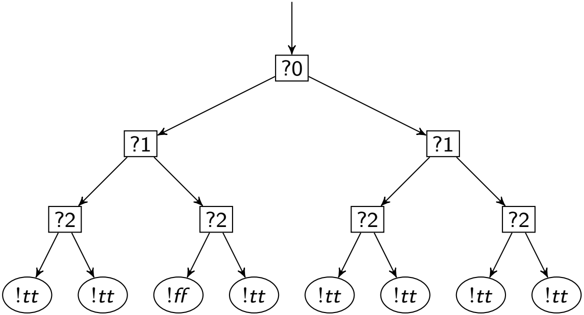
Lemma (No shared computation (B))

If p and p' are distinct points then their subcomputations are disjoint.

Since each subcomputation has length at least $\Omega(n)$ the entire computation must have at least length $\Omega(n2^n)$.

Threads and sections

Consider a 3-standard predicate *seven* (has seven true leaves)



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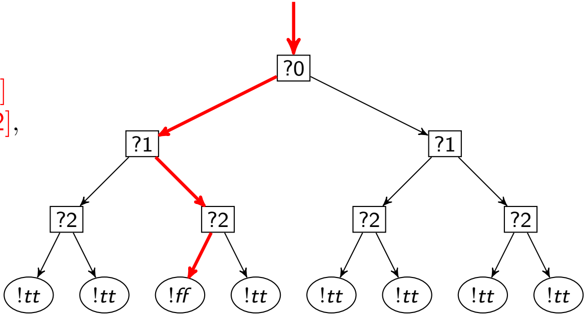
$$\mathit{Thread} \doteq \{\ \mathit{pred}\ p \rightsquigarrow^* \mathcal{E}_0[p\,0],\ \mathcal{E}_0[\mathsf{true}] \rightsquigarrow^* \mathcal{E}_1[p\,1]\ \mathcal{E}_1[\mathsf{false}] \rightsquigarrow^* \mathcal{E}_2[p\,2],\ \mathcal{E}_2[\mathsf{true}] \rightarrow \mathsf{false}\ \}$$



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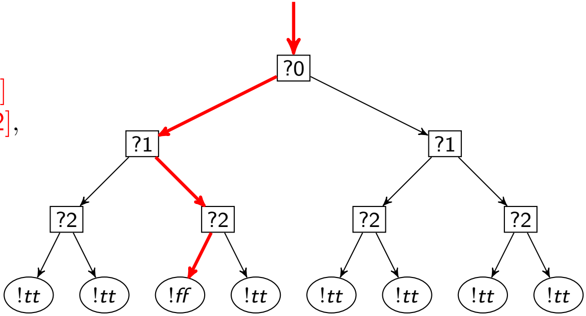

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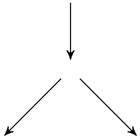
Proof of Lemma A.

By contradiction: pick a leaf that has no thread; negate the value at the leaf; tweak the predicate accordingly; observe a wrong result. $\square$

No shared computation

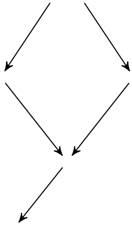
Every section has a unique successor



Proof.

Follows by definition of section and the semantics being deterministic. □

Every section has a single predecessor



Proof.

By direct calculation on the reduction sequence induced by a section. □

Summary and future work

In summary

- We have defined two languages $\mathcal{L}$ and $\mathcal{L}_{\mathit{eff}}$
- We have demonstrated that $\mathcal{L}_{\mathit{eff}}$ provides strictly more efficient implementations of generic search than $\mathcal{L}$ ($\mathcal{O}(2^n)$ vs $\Omega(n2^n)$)
- ...which establish a new complexity result for control operators
- Intuition: control operators build in support for backtracking.

Future considerations

- Perform empirical experiments to observe the result in practice (Daniels 2016)
- Study the robustness of the result, i.e. what feature(s) can we add to $\mathcal{L}$ whilst retaining an efficiency gap between $\mathcal{L}$ and $\mathcal{L}_{\mathit{eff}}$?
- Generalise the result to all conceivable effective models of computations

References I

Daniels, Robbie (Aug. 2016). "Efficient Generic Searches and Programming Language Expressivity". MA thesis. Scotland: School of Informatics, the University of Edinburgh.

Longley, John and Dag Normann (2015). *Higher-Order Computability*. Theory and Applications of Computability. Springer.